Use of Inverse Trigonometric Functions with Electronic Calculators

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1 Introduction

When you use the inverse trigonometric functions on your calculator ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$), your calculator will return an angle as its result. In general, though, your calculator should return *two* answers. This note shows how to find the other angle—the one your calculator doesn't show you.

2 Arcsin

Given a number between -1 and $+1$, the $\sin^{-1}$ (inverse sine) function on your calculator will return an angle θ between $-90°$ and $+90°$. The other angle will be $180° - \theta$.

Example 1. What are the arcsines of 0.224?

Answer 1. Finding $\sin^{-1}(0.224)$ on your calculator (in "degree" mode) returns an answer of $12°.9441$. The other answer is $180° - 12°.9441 = 167°.0559$. On the TI-83+ calculator:

```
 sin⁻¹(0.224) [ENTER]
   12.94408154
180 - [2nd] ANS [ENTER]
   167.0559185
```

Example 2. What are the arcsines of -0.1325?

Answer 2. Finding $\sin^{-1}(-0.1325)$ on your calculator (in "degree" mode) returns an answer of $-7°.6141$. The other answer is $180° - (-7°.6141) = 187°.6141$. On the TI-83+ calculator:

```
 sin⁻¹(-0.1325) [ENTER]
   -7.61408174
180 - [2nd] ANS [ENTER]
   187.6140817
```

1

3 Arccos

Given a number between -1 and $+1$, the $\cos^{-1}$ (inverse cosine) function on your calculator will return an angle θ between $0°$ and $180°$. The other angle will be $-\theta$.

Example 3. What are the arccosines of 0.759?

Answer 3. Finding $\cos^{-1}(0.759)$ on your calculator (in "degree" mode) returns an answer of $40°.6239$. The other answer is $-40°.6239$. (You may add $360°$ to this if you wish; it's the same angle. $-40°.6239 = -40°.6239 + 360° = 319°.3761$.

On the TI-83+ calculator:

```
cos⁻¹(0.759) [ENTER]
  40.62388077
(-) [2nd] ANS [ENTER]
  -40.62388077
[2nd] ANS + 360 [ENTER]
  319.3761192
```

Example 4. What are the arccosines of -0.344 ?

Answer 4. Finding $\cos^{-1}(-0.344)$ on your calculator (in "degree" mode) returns an answer of $110°.1208$. The other answer is $-110°.1208$. (You may add $360°$ to this if you wish; it's the same angle. $-110°.1208 = -110°.1208 + 360° = 249°.8792$.

On the TI-83+ calculator:

```
cos⁻¹(-0.344) [ENTER]
  110.120764
(-) [2nd] ANS [ENTER]
  -110.120764
[2nd] ANS + 360 [ENTER]
  249.879236
```

4 Arctan

Given a real number, the $\tan^{-1}$ (inverse tangent) function on your calculator will return an angle θ between $-90°$ and $+90°$. The other angle will be $180° + \theta$.

We often need to take the inverse tangent of a *ratio* of two numbers. In this case, you can place the inverse tangent in the correct quadrant by adding $180°$ to the calculator's result if the denominator of the ratio is negative.

Some computer programming languages include *two* arctangent functions: one that takes one argument (like a calculator), and another that takes the numerator and denominator of a ratio. For example, C has two such functions, called `atan()` and `atan2()`. Using `atan2(y,x)` computes $\tan^{-1}(y/x)$ and returns an angle in the correct quadrant.

Example 5. What are the arctangents of 1.784 ?

Answer 5. Finding $\tan^{-1}(1.784)$ on your calculator (in "degree" mode) returns an answer of $60°.7277$. The other answer is $60°.7277 + 180° = 240°.7277$.

On the TI-83+ calculator:

```
tan⁻¹(1.784) [ENTER]
   60.72770782
[2nd] ANS + 180 [ENTER]
   240.7277078
```

Example 6. What are the arctangents of -0.821 ?

Answer 6. Finding $\tan^{-1}(-0.821)$ on your calculator (in "degree" mode) returns an answer of $-39°.3860$. The other answer is $-39°.3860 + 180° = 140°.6140$. On the TI-83+ calculator:

```
tan⁻¹(-0.821) [ENTER]
   -39.38599545
[2nd] ANS + 180 [ENTER]
   140.6140046
```

Example 7. What is the arctangent of $(-4)/3$?

Answer 7. Finding $\tan^{-1}(-4/3)$ on your calculator (in "degree" mode) returns an answer of $-53°.1301$. Since the denominator (3) is positive, we do nothing more; this is the final answer. On the TI-83+ calculator:

```
tan⁻¹(-4/3) [ENTER]
   -53.13010235
```

Example 8. What is the arctangent of $4/(-3)$?

Answer 8. Finding $\tan^{-1}(-4/3)$ on your calculator (in "degree" mode) returns an answer of $-53°.1301$. Since the denominator (-3) is negative, we add $180°$: $-53°.1301 + 180° = 126°.8699$. On the TI-83+ calculator:

```
tan⁻¹(-4/3) + 180 [ENTER]
   126.8698976
```

5   Summary

The following table summarizes how to find both inverse trigonometric function results with your calculator:

	Calculator answer	Other answer
$\sin^{-1}$	θ	$180° - \theta$
$\cos^{-1}$	θ	$-\theta$
$\tan^{-1}$	θ	$\theta + 180°$

When computing $\tan^{-1}(y/x)$, add $180°$ to the calculator's answer if the denominator (x) is negative.

Multiples of $360°$ may always be added to or subtracted from an angle; the result is the same angle.