Technical Physics

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Dedication

For the late Dr. Michael R. Collier, who formerly taught Technical Physics at Prince George's Community College. Dr. Collier set a high standard of excellence in both teaching and in scientific research at NASA that continues to be an inspiration to me, to his former students, and to his scientific colleagues.

Contents

Chapter 1

Introduction

TECHNICAL physics is not a well-defined area of physics. But broadly speaking, it is meant to cover the major ideas of classical physics — mechanics, acoustics, thermodynamics, electromagnetism, and optics — with some emphasis on applications. The student should have a strong background in algebra and trigonometry. If a student feels he is weak in one or more areas of mathematics or calculator skills, a supplementary set of course notes available for this course, *Introduction to College Mathematics* by the author of these notes, and available at this course's Web site. No knowledge of the calculus is assumed, nor is calculus used in this text. For a calculus-based course, the student is referred to the General Physics sequence (PHY 2020, 2030, 2040) offered by the college.

In addition to technical physics, this course will cover a bit about *remote sensing*: the detection and measurement of the physical characteristics of an area (typically on the Earth) by measuring its reflected and emitted radiation from a distance. This is as opposed to *in situ* measurement, in which measurements are made at the actual location of interest. Remote sensing these days is often performed with orbiting satellites.

This text begins with a discussion of the ancient Greek alphabet (which is heavily used in science, mathematics, and engineering), followed by a review of some mathematics. The main part of the course comes next — Part III, covering a number of areas of technical physics. Part IV discusses remote sensing — a technique used by many Earth-observing satellites to make measurements of the Earth. Finally, a brief Part V covers some thoughts about professional ethics.

There are some things that would be useful to commit to memory: the Greek alphabet (so that you know their letters by their proper name; Appendix 3); and the SI prefixes, at least through $10^{\pm 18}$ (exa- through atto-; Table 20-3 in Appendix 20), so that you know what power of 10 each prefix represents without having to look it up.

We can hope to give only a brief overview of some of the key ideas in each of these fields. There is much, much more to each of these topics than can be presented here. But it is the author's hope that at least the student will take away some of the key concepts of classical physics, perhaps as a starting point to learn these subjects in greater depth.

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Part I

Ancient Greek

Chapter 2

The Greek Alphabet

Ancient Greek is the language spoken in the region around modern Greece, from time of the poet Homer (8th century BC) through the time of classical Greece (4th and 5th centuries BC). Its alphabet consists of 24 letters, as shown in Table 2-1.

This is not a course in ancient Greek, so we will not have the opportunity to study the language in detail. However, the letters of the Greek alphabet are very common in physics, mathematics, and engineering, because we need symbols to stand for different quantities, and the 26 letters of the Roman/English alphabet are often not enough.[1]

Table 2-1 shows the 24 letters of the Greek alphabet. In order to be mathematically and scientifically literate, and so that you can communicate with others who study mathematics and physical science, you should know the proper names of these Greek letters. Therefore you should *memorize the entire Greek alphabet*, so that you can provide the correct name of each letter when you see it.[2] The table shows both uppercase and lowercase forms are shown in the table.

Note that several letters have alternate forms. Especially common are the two forms for *epsilon* (ϵ and ε) and for *phi* (ϕ and φ).

Note also that there are *two* different forms of the Greek lowercase letter *sigma*. The second form (ς) is used when *sigma* is the last letter of a word; otherwise the form σ is used.[3] This variation affects *only* lowercase *sigma*, not its uppercase counterpart, which is always written as Σ. Although common in ancient Greek writing, the ς form of the letter *sigma* is almost never used in mathematics, physics, or engineering.

Originally the Greeks wrote using only capital letters. There was no punctuation, nor were there even spaces between words. Lowercase letters, spacing, and punctuation were later additions to the language, and are used today when printing and reading ancient Greek texts.[4] Unlike in English, the first letter of a Greek sentence is generally *not* capitalized; only proper names begin with a capital letter.

Before the time of Homer, during the Greek Dark Ages (1200–800 BC), the Greek language was written using a completely different writing system—a syllabary called Linear B, which was not deciphered until the 1950s. [1,2]

A mnemonic device for remembering the numbering of the Greek letters is EKOΥ: these are the 5th, 10th, 15th, and 20th letters of the Greek alphabet.

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[1] Rarely, authors will sometimes even dip into the Hebrew alphabet when even the Roman and Greek alphabets together don't provide enough symbols.

[2] By tradition, in science and mathematics we call Greek letters by their ancient Greek names. Their names in modern Greek are similar, but often different.

[3] English had a similar feature until fairly recently. Until the 19th century, the letter *s* was written as a "long s" symbol ($\int$) when it was in the middle of a word. You'll see this usage in some older documents like the Declaration of Independence.

[4] These statements are also true of ancient Latin: originally there were no spaces, punctuation, or lowercase letters.

Table 2-1. The Greek alphabet.

Letter	Name	Pronunciation
Α α	alpha	*a* as in *father*
Β β	beta	*b* as in *boy*
Γ γ	gamma	always hard *g*, as in *girl*
Δ δ	delta	*d* as in *dent*
Ε ε	epsilon	*e* as in *get*
Ζ ζ	zeta	*dz* as in *adze*
Η η	eta	*ay* as in *hay*
Θ θ	theta	unvoiced *th* as in *thorn*
Ι ι	iota	*i* as in *pin* (unstressed) or *machine* (stressed)
Κ κ	kappa	*k* as in *kill*
Λ λ	lambda	*l* as in *lamb*
Μ μ	mu	*m* as in *money*
Ν ν	nu	*n* as in *nest*
Ξ ξ	xi	*x* or *ks*, as in *box*
Ο ο	omicron	*o* as in *got*
Π π	pi	*p* as in *pie*
Ρ ρ	rho	*r* trilled or flipped as in Spanish *pero*
Σ σ ς	sigma	*s* as in *song*
Τ τ	tau	*t* as in *tank*
Υ υ	upsilon	*u* as in *tube*
Φ φ	phi	*f* or *ph* as in *phase*
Χ χ	chi	as in Scottish *loch*; or like English *k*
Ψ ψ	psi	*ps* as in *oops*
Ω ω	omega	*o* as in *poke*

(Alternate forms: ϐ $= \beta$, $\epsilon = \varepsilon$, $\vartheta = \theta$, $\varkappa = \kappa$, $\varpi = \pi$, $\varrho = \rho$, $\varsigma = \sigma$, Y$= \Upsilon$, $\phi = \varphi$.)

Chapter 3

Quotations from Classical Greek

Here are some notable quotations from ancient Greek authors and philosophers. [7] As practice with the Greek alphabet, try spelling each word by providing the name of each letter. You may also try pronouncing each quotation. Can you spot any Greek words that might be the source of words in English? (For example, the Greek word *πεντε* means *five*, and is the source of the English word *pentagon* (a five-sided polygon).)

το γαρ εξαπατασθαι αυτον υφ αυτου παντων χαλεπωτατον.
The worst of all deceptions is self-deception. —Plato

ηθος ανθρωπω δαιμων.
A man's character is his fate. —Heraclitus

ο δε ανεξεταστος βιος ου βιωτος ανθρωπω.
An unexamined life is not worth living. —Socrates

δος μοι πα στω και ταν γαν κινησω.
Give me but one firm spot on which to stand, and I will move the world. —Archimedes, describing the lever

ευρηκα, ευρηκα!
Eureka! I have discovered it! —Archimedes, jumping out of a public bath as he discovered the laws of displacement

κτηματων παντων εστι τιμιωτατον ανηρ φιλος συνετος τε και ευνοος.
The most precious of all possessions is a wise and loyal friend. —Herodotus

ο μεν ουν δειλος και α μη δει φοβειται.
A coward fears even things he ought not to fear. —Aristotle

μετρον δ′ επι πασιν αριστον.
Moderation in all things is best. —Pythagoras
(The Roman philosopher Seneca later expressed the same sentiment in Latin: *Modum tenēre dēbēmus.*)

Part II

Physics

Chapter 4

Introduction to Physics

Physics is the most fundamental of the sciences. Its goal is to learn how the Universe works at the most fundamental level—and to discover the basic laws by which it operates. *Theoretical physics* concentrates on developing the theory and mathematics of these laws, while *applied physics* focuses attention on the application of the principles of physics to practical problems. *Experimental physics* lies at the intersection of physics and engineering; experimental physicists have the theoretical knowledge of theoretical physicists, *and* they know how to build and work with scientific equipment.

Physics is divided into a number of sub-fields, and physicists are trained to have some expertise in all of them. This variety is what makes physics one of the most interesting of the sciences—and it makes people with physics training very versatile in their ability to do work in many different technical fields.

The major fields of physics are:

- *Classical mechanics* is the study the motion of bodies according to Newton's laws of motion.
- *Electricity and magnetism* are two closely related phenomena that are together considered a single field of physics.
- *Quantum mechanics* describes the peculiar motion of very small bodies (atomic sizes and smaller).
- *Optics* is the study of light.
- *Acoustics* is the study of sound.
- *Thermodynamics* and *statistical mechanics* are closely related fields that study the nature of heat.
- *Solid-state physics* is the study of solids—most often crystalline metals.
- *Plasma physics* is the study of plasmas (ionized gases).
- *Atomic, nuclear, and particle physics* study of the atom, the atomic nucleus, and the particles that make up the atom.
- *Relativity* includes Albert Einstein's theories of special and general relativity. *Special relativity* describes the motion of bodies moving at very high speeds (near the speed of light), while *general relativity* is Einstein's theory of gravity.

We'll not be able to cover all of these subjects in this course, but we will get an overview of some of the major ideas in physics and how they are applied to real-world problems.

The fields of *cross-disciplinary physics* combine physics with other sciences. These include *astrophysics* (physics of astronomy), *geophysics* (physics of geology), *biophysics* (physics of biology), *chemical physics* (physics of chemistry), and *mathematical physics* (mathematical theories related to physics).

Besides acquiring a knowledge of physics for its own sake, the study of physics will give you a broad technical background and set of problem-solving skills that you can apply to wide variety of other fields. Some students of physics go on to study more advanced physics, while others find ways to apply their knowledge of physics to such diverse subjects as mathematics, engineering, biology, medicine, and finance.

Another benefit of learning physics is that, unlike courses in technology, everything you learn in this course will never be obsolete. Although theories at the cutting edge of physics research may change, the basic physics you'll learn in this course will not. You will be able to use what you learn in this course throughout your life.

Chapter 5

Mathematics Review

5.1 Scientific Notation

Scientific notation is a convenient way to write very large or very small numbers, without having to write (or count) lots of zeros.

Powers of 10

We begin with a discussion of powers of 10, written as 10^n, where (for now) $n > 1$. This means 1 followed by n zeros. For example:

10^1	= 10	ten
10^2	= 100	one hundred
10^3	= 1000	one thousand
10^4	= 10,000	ten thousand
10^5	= 100,000	one hundred thousand
10^6	= 1,000,000	one million
10^{34}	= 10,000,000,000,000,000,000,000,000,000,000,000	ten decillion

Clearly the power-of-10 notation is a much more compact way of writing these numbers, especially for very large numbers. Furthermore, it avoids having to write and count all those zeros.

This power-of-10 notation may be used for very small numbers as well: the notation 10^{-n} ($n > 1$) means a zero, then a decimal point, then $n - 1$ zeros, followed by a 1. For example:

10^{-1}	= 0.1	one tenth
10^{-2}	= 0.01	one one hundredth
10^{-3}	= 0.001	one one thousandth
10^{-4}	= 0.0001	one ten thousandth
10^{-5}	= 0.000 01	one one hundred thousandth
10^{-6}	= 0.000 001	one one millionth
10^{-34}	= 0.000 000 000 000 000 000 000 000 000 000 000 1	one ten decillionth

Again, it's clear that the power-of-ten notation compact and saves writing and counting a lot of zeros for very small numbers.

One case we've not mentioned yet is 10^0, which is simply equal to 1; *any* number to the power 0 is equal to 1.

In summary: for positive integers n:

- 10^n is 1 followed by n zeros.
- 10^{-n} is "0.", followed by $n-1$ zeros, followed by 1.
- $10^0 = 1$.

Scientific Notation for Other Numbers

So far we've seen how to write even powers of 10 using exponents. In fact, *any* real number may be written as an ordinary real number multiplied by some appropriate power of 10. One very simple way to do this is to simply multiply the original number by 10^0; since $10^0 = 1$, this is just multiplying by 1 and does not change the number. For example:

34.5	may be written 34.5×10^0
778.6	may be written 778.6×10^0
0.054	may be written 0.054×10^0
21,900	may be written $21{,}900 \times 10^0$

While the numbers on the right are indeed in "scientific notation" (a real number multiplied by a power of 10), the usual convention is to have the number multiplying the power of 10 be a value between 1 and 10. So "standard" scientific notation is:

$$r \times 10^n \qquad (1 \le r < 10)$$

We can convert any number to this range by following this rule: *multiply r by 10^m, then divide 10^n by 10^m*, for some appropriate integer m. By multiplying and dividing by the same number, we haven't changed the number. For example, take the first number above: 34.5×10^0. We can make the number in front (34.5) fall between 1 and 10 by dividing it by 10; to compensate and leave the number unchanged, we'll *multiply* the power of 10 by 10. Thus:

$$\begin{aligned} 34.5 \times 10^0 &= (34.5/10) \times (10^0 \times 10) \\ &= 3.45 \times 10^1 \end{aligned}$$

and this is the original number in standard scientific notation. When manipulating the powers of 10, remember the rule for exponents: when multiplying like bases, we add exponents. In this case, we did

$$10^0 \times 10 = 10^0 \times 10^1 = 10^{0+1} = 10^1$$

Now let's look at the second number above, 778.6×10^0. We can put this in standard scientific notation by dividing the number in front (778.6) by 100; we then compensate by multiplying the power of 10 by 100 to keep the original number the same:

$$\begin{aligned} 778.6 \times 10^0 &= (778.6/100) \times (10^0 \times 100) \\ &= 7.786 \times (10^0 \times 10^2) \\ &= 7.786 \times 10^{0+2} \\ &= 7.786 \times 10^2 \end{aligned}$$

This is the original number in scientific notation.

Now let's try the third example above, which is less than 1: 0.054×10^0. To put the number 0.054 in the range between 1 and 10, we'll have to move the decimal point two places to the right, i.e. multiply by $100 = 10^2$. We then compensate by *dividing* the power of 10 by 100 to keep the original number unchanged:

$$\begin{aligned} 0.054 \times 10^0 &= (0.054 \times 100) \times (10^0/10^2) \\ &= 5.4 \times 10^{0-2} \\ &= 5.4 \times 10^{-2} \end{aligned}$$

This is the original number in standard scientific notation. (Remember that when dividing like bases, we subtract exponents.)

Finally, let's try the last example above: $21{,}900 \times 10^0$. To get the number in front (21,900) between 1 and 10, we'll have to move the decimal point 4 places to the left; in other words, divide by 10^4. We'll compensate by multiplying the power of 10 by 10^4:

$$\begin{aligned} 21{,}900 \times 10^0 &= (21{,}900/10^4) \times (10^0 \times 10^4) \\ &= 2.1900 \times 10^{0+4} \\ &= 2.1900 \times 10^4 \end{aligned}$$

This is the original number in standard scientific notation.

Some Thoughts

One reason scientific notation is useful is that we very rarely have need to carry lots of significant digits around in a calculation — most measurements are just not known that extremely high accuracy. In a previous example, we saw the number

451,821,003,405,111,818,321

There are *very* few measurements or calculations in the physical world that require that many significant digits. Usually rounding this number to 4.5182×10^{20}, with five significant digits, would be sufficient, and saves a lot of space. You certainly don't save any space calling this number $4.518\,210\,034\,051\,118\,183\,21 \times 10^{20}$, but then you would practically never be able to measure, or need to know, that many significant digits.

Another reason for using scientific notation is that it allows us to quickly judge the relative sizes (magnitudes) of numbers. We need only look at the exponents; we don't need to spend time counting the number of digits present.

Scientific Notation on a Scientific Calculator

Electronic scientific calculators are designed to allow entry of numbers in scientific notation.

TI 84 Plus Calculator

On the TI 84 Plus calculator, we use the key sequence `2nd` `EE` (above the comma key) to enter the power of 10. For example, to enter the number 3.57×10^8, we would enter:

`3` `.` `5` `7` `2nd` `EE` `8`

Do *not* use the exponent key [^] or the anti-logarithm function [10^x] to enter numbers in scientific notation. *Always* use [EE]. The other keys are for raising number to powers, not for entering numbers in scientific notation.

For negative numbers, for example -3.57×10^{-8}, we would enter:

[(−)] [3] [.] [5] [7] [2nd] [EE] [(−)] [8]

The TI 84 Plus calculator includes a special shortcut for entering powers of 10: just enter [EE] followed by the exponent. For example, to enter $10^7 = 1.0 \times 10^7$, one may simply enter

[2nd] [EE] [7]

HP 42S Calculator

On the HP 42S calculator (RPN mode), we use the key [EEX] to enter the power of 10. For example, to enter the number 3.57×10^8, we would enter:

[3] [.] [5] [7] [EEX] [8]

Do *not* use the exponent function [y^x] or the anti-logarithm function [10^x] to enter numbers in scientific notation. *Always* use [EEX]. The other keys are for raising number to powers, not for entering numbers in scientific notation.

For negative numbers, for example -3.57×10^{-8}, we would enter:

[3] [.] [5] [7] [+/−] [EEX] [8] [+/−]

Hewlett-Packard RPN calculators include a special shortcut for entering powers of 10: just enter [EEX] followed by the exponent. For example, to enter $10^7 = 1.0 \times 10^7$, one may simply enter

[EEX] [7]

This enters 1.0×10^7 on the stack.

5.2 Length

5.3 Area and Volume

Chapter 6

Units

The phenomena of Nature have been found to obey certain physical laws; one of the primary goals of physics research is to discover those laws. It has been known for several centuries that the laws of physics are appropriately expressed in the language of *mathematics*, so physics and mathematics have enjoyed a close connection for quite a long time.

In order to connect the physical world to the mathematical world, we need to make *measurements* of the real world. In making a measurement, we compare a physical quantity with some agreed-upon standard, and determine how many such standard units are present. For example, we have a precise definition of a unit of length called a *mile*, and have determined that there are about 92,000,000 such miles between the Earth and the Sun.

It is important that we have very precise definitions of physical units — not only for scientific use, but also for trade and commerce. In practice, we define a few *base units*, and derive other units from combinations of those base units. For example, if we define units for length and time, then we can define a unit for speed as the length divided by time (e.g. miles/hour).

How many base units do we need to define? There is no magic number; in fact it is possible to define a system of units using only *one* base unit (and this is in fact done for so-called *natural units*). For most systems of units, it is convenient to define base units for length, mass, and time; a base electrical unit may also be defined, along with a few lesser-used base units.

6.1 Systems of Units

Several different systems of units are in common use. For everyday civil use, most of the world uses *metric* units. The United Kingdom uses both metric units and an *imperial* system. Here in the United States, *U.S. customary units* are most common for everyday use.[1]

There are actually several "metric" systems in use. They can be broadly grouped into two categories: those that use the meter, kilogram, and second as base units (MKS systems), and those that use the centimeter, gram, and second as base units (CGS systems). There is only one MKS system, called *SI units*. We will mostly use SI units in this course, but we will use other systems from time to time so that you get some experience with using them.

[1] In the mid-1970s the U.S. government attempted to switch the United States to the metric system, but the idea was abandoned after strong public opposition. One remnant from that era is the two-liter bottle of soda pop.

6.2 SI Units

SI units (which stands for Système International d'unités) are based on the *meter* as the base unit of length, the *kilogram* as the base unit of mass, and the *second* as the base unit of time. SI units also define four other base units (the *ampere*, *kelvin*, *candela*, and *mole*, to be described later). Any physical quantity that can be measured can be expressed in terms of these seven base units or some combination of them. SI units are summarized in Appendix 20.

SI units were originally based mostly on the properties of the Earth and of water. Under the *original* definitions:

- The *meter* was defined to be one ten-millionth the distance from the equator to the North Pole, along a line of longitude passing through Paris.
- The *kilogram* was defined as the mass of 0.001 m^3 of water.
- The *second* was defined as 1/86,400 the length of a day (one rotation of the Earth).
- The definition of the *ampere* is related to electrical properties, ultimately relating to the meter, kilogram, and second.
- The *kelvin* was defined in terms of the thermodynamic properties of water, as well as absolute zero.
- The *candela* was defined by the luminous properties of molten tungsten and the behavior of the human eye.
- The *mole* was defined by the density of the carbon-12 nucleus.

Many of these original definitions have been replaced over time with more precise definitions, as the need for increased precision has arisen. Most recently, on May 20, 2019, there was a major re-definition of SI units, in which the definitions of the kilogram, ampere, kelvin, and mole were all changed. SI units now really have only one unit that is determined experimentally: the unit of time, which is the *second*. The other base units are now defined by defining exact, unchanging values for several of the physical constants.

Length (Meter)

The SI base unit of length, the *meter* (m), has been re-defined more times than any other unit, due to the need for increasing accuracy. Originally (1793) the meter was defined to be 1/10,000,000 the distance from the North Pole to the equator, along a line going through Paris.[2] Then, in 1889, the meter was re-defined to be the distance between two lines engraved on a prototype meter bar kept in Paris. Then in 1960 it was re-defined again: the meter was defined as the distance of 1,650,763.73 wavelengths of the orange-red emission line in the krypton-86 atomic spectrum. Still more stringent accuracy requirements led to the the current definition of the meter, which was implemented in 1983: the meter is now defined to be the distance light in vacuum travels in 1/299,792,458 second. Because of this definition, the speed of light is now *exactly* 299,792,458 m/s.

U.S. Customary units are legally defined in terms of metric equivalents. For length, the *foot* (ft) is defined to be exactly 0.3048 meter.

[2] If you remember this original definition, then you can remember the circumference of the Earth: about 40,000,000 meters.

Mass (Kilogram)

Originally the *kilogram* (kg) was defined to be the mass of 1 liter (0.001 m^3) of water. The need for more accuracy required the kilogram to be re-defined to be the mass of a standard mass called the *International Prototype Kilogram* which is kept in a vault at the Bureau International des Poids et Mesures (BIPM) in Paris. Each country was given its own copy of the IPK to use as its own national standard.

In 2019, the kilogram was re-defined (somewhat indirectly) by defining *Planck's constant* (used in quantum mechanics) to be *exactly* equal to $h = 6.62607015 \times 10^{-34}$ kg m^2 s^{-1}. Since the meter and second are given precise experimental definitions, fixing the value of h has the effect of defining the value for the kilogram.

Another common metric (but non-SI) unit of mass is the *metric ton*, which is 1000 kg (a little over 1 short ton).

In U.S. customary units, the *pound-mass* (lbm) is defined to be exactly 0.45359237 kg.

Mass vs. Weight

Mass is not the same thing as *weight*, so it's important not to confuse the two. The *mass* of a body is a measure of the total amount of matter it contains; the *weight* of a body is the gravitational force on it due to the Earth's gravity. At the surface of the Earth, mass m and weight W are proportional to each other:

$$W = mg, \tag{6.1}$$

where g is the acceleration due to the Earth's gravity, equal to 9.80 m/s^2. Remember: mass is mass, and is measured in kilograms; weight is a force, and is measured in force units of *newtons*.

Time (Second)

Originally the base SI unit of time, the *second* (s), was defined to be $1/60$ of $1/60$ of $1/24$ of the length of a day, so that 60 seconds = 1 minute, 60 minutes = 1 hour, and 24 hours = 1 day. High-precision time measurements have shown that the Earth's rotation rate has short-term irregularities, along with a long-term slowing due to tidal forces. So for a more accurate definition, in 1967 the second was re-defined to be based on a definition using atomic clocks. The second is now defined to be the time required for 9,192,631,770 oscillations of a certain type of radiation emitted from a cesium-133 atom.

Although officially the symbol for the second is "s", you will also often see people use "sec" to avoid confusing lowercase "s" with the number "5".

The Ampere, Kelvin, and Candela

For this course, most quantities will be defined entirely in terms of meters, kilograms, and seconds. There are four other SI base units, though: the *ampere* (A) (the base unit of electric current); the *kelvin* (K) (the base unit of temperature); the *candela* (cd) (the base unit of luminous intensity, or light brightness); and the *mole* (mol) (the base unit of amount of substance). With the 2019 re-definition of SI units, the *ampere* is now defined by fixing the value of the elementary charge to *exactly* $e = 1.602176634 \times 10^{-19}$ A s. The *kelvin* is now defined by fixing the value of *Boltzmann's constant* to *exactly* $k_B = 1.380649 \times 10^{-23}$ J/K. The *candela* is a unit that measures the brightness of light, and has a somewhat complex defintion that includes a model of the response of the human eye to light of different wavelengths.

Amount of Substance (Mole)

Since we may have a use for the mole in this course, let's look at its definition in detail. The simplest way to think of it is as the name for a number. Just as "thousand" means 1,000, "million" means 1,000,000, and "billion" means 1,000,000,000, in the same way "mole" refers to the number[3] 602,214,076,000,000,000,000,000, or $6.02214076 \times 10^{23}$. You could have a mole of grains of sand or a mole of Volkswagens, but most often the mole is used to count atoms or molecules. There is a reason this number is particularly useful: since each nucleon (proton and neutron) in an atomic nucleus has an average mass of $1.66053906660 \times 10^{-24}$ grams (called an *atomic mass unit*, or amu), then there are $1/(1.66053906660 \times 10^{-24})$, or $6.02214076 \times 10^{23}$ nucleons per gram. In other words, one mole of nucleons has a mass of 1 gram. Therefore, if A is the atomic weight of an atom, then A moles of nucleons has a mass of A grams. But A moles of nucleons is the same as 1 mole of atoms, so *one mole of atoms has a mass (in grams) equal to the atomic weight*. In other words,

$$\text{moles of atoms} = \frac{\text{grams}}{\text{atomic weight}} \tag{6.2}$$

Similarly, when counting molecules,

$$\text{moles of molecules} = \frac{\text{grams}}{\text{molecular weight}} \tag{6.3}$$

In short, the mole is useful when you need to convert between the mass of a material and the number of atoms or molecules it contains.

It's important to be clear about what exactly you're counting (atoms or molecules) when using moles. It doesn't really make sense to talk about "a mole of oxygen", any more than it would be to talk about "100 of oxygen". It's either a "mole of oxygen atoms" or a "mole of oxygen molecules".[4]

For convenience, sometimes the word *entity* is used to mean "atom or molecule." Then the formula for determining the number of moles from the mass becomes

$$\text{moles of entities} = \frac{\text{grams}}{\text{entity weight}} \tag{6.4}$$

where *entity weight* means either atomic weight or molecular weight, depending on whether it's atoms or molecules that are being discussed.

Note that although the base SI unit of mass is the *kilogram*, the mole is defined by having the number of *grams* equal to the entity weight. Other kinds of "moles" have been defined, such as the *pound-mole*, *ounce-mole*, and *kilogram-mole*, in which the indicated unit of mass is numerically equal to the entity weight. For example, 1 kilogram-mole of carbon-12 atoms is 12 kilograms of carbon-12, and contains $6.02214076 \times 10^{26}$ carbon atoms. The SI mole is the same things as a gram-mole.

With the 2019 SI units re-definition, the mole is defined by setting Avogadro's constant equal to *exactly* $N_A = 6.02214076 \times 10^{23}\ \text{mol}^{-1}$.

Interesting fact: it's estimated that there is roughly one mole of stars in the observable Universe.

SI Derived Units

In addition to the seven base units (m, kg, s, A, K, cd, mol), there are a number of so-called *SI derived units* with special names. We'll introduce these as needed, but a summary of all of them is shown in Appendix 20 (Table 20-2). These are just combinations of base units that occur often enough that it's convenient to give them special names.

Plane Angle (Radian)

One derived SI unit that we will encounter frequently is the SI unit of plane angle. Plane angles are commonly measured in one of two units: *degrees* or *radians*.[5] You're probably familiar with degrees already: one full

[3] Six hundred two sextillion, two hundred fourteen quintillion, seventy-six quadrillion.

[4] Sometimes chemists will refer to a "mole of oxygen" when it's understood whether the oxygen in question is in the atomic (O) or molecular (O_2) state.

[5] A third unit implemented in many calculators is the *grad*: a right angle is 100 grads and a full circle is 400 grads. You may encounter grads in some older literature, such as Laplace's *Mécanique Céleste*. Almost nobody uses grads today, though.

circle is 360°, a semicircle is 180°, and a right angle is 90°.

The SI unit of plane angle is the *radian*, which is defined to be that plane angle whose arc length is equal to its radius. This means that a full circle is 2π radians, a semicircle is π radians, and a right angle is $\pi/2$ radians. To convert between degrees and radians, then, we have:

$$\text{degrees} = \text{radians} \times \frac{180}{\pi} \tag{6.5}$$

and

$$\text{radians} = \text{degrees} \times \frac{\pi}{180} \tag{6.6}$$

The easy way to remember these formulæ is to think in terms of units: 180 has units of degrees and π has units of radians, so in the first equation units of radians cancel on the right-hand side to leave degrees, and in the second equation units of degrees cancel on the right-hand side to leave radians.

Occasionally you will see a formula that involves a "bare" angle that is not the argument of a trigonometric function like the sine, cosine, or tangent. In such cases it is understood that the angle must be *in radians*. For example, the radius of a circle r, angle θ, and arc length s are related by

$$s = r\theta, \tag{6.7}$$

where it is understood that θ is in radians.

See Appendix 26 for a further discussion of plane and solid angles.

SI Prefixes

It's often convenient to define both large and small units that measure the same thing. For example, in English units, it's convenient to measure small lengths in inches and large lengths in miles.

In SI units, larger and smaller units are defined in a systematic way by the use of *prefixes* to the SI base or derived units. For example, the base SI unit of length is the meter (m), but small lengths may also be measured in centimeters (cm, 0.01 m), and large lengths may be measured in kilometers (km, 1000 m). Table 20-3 in Appendix 20 shows all the SI prefixes and the powers of 10 they represent. You should *memorize* the powers of 10 for all the SI prefixes in this table.

To use the SI prefixes, simply add the prefix to the front of the name of the SI base or derived unit. The symbol for the prefixed unit is the symbol for the prefix written in front of the symbol for the unit. For example, kilometer (km) $= 10^3$ meter, microsecond (μs) $= 10^{-6}$ s. But put the prefix on the *gram* (g), *not* the kilogram: for example, 1 microgram (μg) $= 10^{-6}$ g. For historical reasons, the kilogram is the only SI base or derived unit with a prefix.[6]

The 2019 Re-definition of SI Units

On May 20, 2019, a major re-definition of SI units went into effect. With this re-definition, experimental definitions of several of the SI units have been replaced by *defining* the values of several fundamental physical constants, so that these values become fixed and unchanging, no matter how many future experiments are performed. The defined constants are shown in Table 6-1.

[6] Originally, the metric standard of mass was a unit called the *grave* (*GRAH-veh*), equal to 1000 grams. When the metric system was first established by Louis XVI following the French Revolution, the name *grave* was considered politically incorrect, since it resembled the German word *Graf*, or "Count" — a title of nobility, at a time when titles of nobility were shunned. The *grave* was retained as the unit of mass, but under the more acceptable name *kilogram*. The gram itself was too small to be practical as a mass standard.

Table 6-1. New SI base quantities, defining constants, and definitions.

Base quantity	Defining constant	Definition	Defines SI unit
Frequency	$\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$	The unperturbed ground-state hyperfine splitting frequency of the cesium-133 atom is exactly 9,192,631,770 Hz.	s
Velocity	c	The speed of light in vacuum c is exactly 299,792,458 m/s.	m
Action	h	The Planck constant h is exactly $6.62607015 \times 10^{-34}$ J s.	kg
Electric charge	e	The elementary charge e is exactly $1.602176634 \times 10^{-19}$ C.	A
Heat capacity	k_B	The Boltzmann constant k_B is exactly 1.380649×10^{-23} J/K.	K
Amount of substance	N_A	The Avogadro constant N_A is exactly $6.02214076 \times 10^{23}$ mol^{-1}.	mol
Luminous intensity	K_{cd}	The luminous efficacy K_{cd} of monochromatic radiation of frequency 540×10^{12} Hz is exactly 683 lm/W.	cd

6.3 CGS Systems of Units

In some fields of physics (e.g. solid-state physics, plasma physics, and astrophysics), it has been customary to use CGS units rather than SI units, so you may encounter them occasionally. There are several different CGS systems in use: *electrostatic*, *electromagnetic*, *Gaussian*, and *Heaviside-Lorentz* units. These systems differ in how they define their electric and magnetic units. Unlike SI units, none of these CGS systems defines a base electrical unit, so electric and magnetic units are all derived units. The most common of these CGS systems is Gaussian units, which are summarized in Appendix 21.

SI prefixes are used with CGS units in the same way they're used with SI units.

6.4 British Engineering Units

Another system of units that is common in some fields of engineering is *British engineering units*. In this system, the base unit of length is the foot (ft), and the base unit of time is the second (s). The base unit of force is called the *pound-force* (lbf), and mass is measured units of *slugs*, where 1 slug has a weight of 32.17404855 lbf.

A related unit of mass (not part of the British engineering system) is called the pound-mass (lbm). At the surface of the Earth, a mass of 1 lbm has a weight of 1 lbf, so sometimes the two are loosely used interchangeably and called the *pound* (lb), as we do every day when we speak of weights in pounds.

SI prefixes are not used in the British engineering system.

6.5 Avoirdupois Units

In the United States, weights are commonly measured in *avoirdupois units*.[7] An *ounce* (oz) is 28.34952 grams. Sixteen (16) ounces makes one *pound* (lb) (453.5924 grams), and 2000 pounds makes one (short) ton (0.90718 metric tons).[8] For very small weights, sometimes the *grain* is used: 1 grain is 64.79891 milligrams. One pound is 16 ounces, and 1 ounce is 437.5 grains (exactly). One pound avoirdupois is 7000 grains.

6.6 Liquid Units

Customary units for liquids include the U.S. gallon (exactly 231 cubic inches). The gallon is divided into four *quarts*; the quart into two *pints*; the pint into two *cups*; the cup into 16 *tablespoons*; and the tablespoon into 3 *teaspoons*. Cups, tablespoons, and teaspoons are very common in recipes.

[7] Pronounced (oddly) as *ă-ver-duh-POIZ*.

[8] A *metric ton*, or *tonne*, is 1000 kg.

Furthermore, one may divide the cup into 8 *fluid ounces*, where 1 fluid ounce is 29.57353 cubic centimeters. One fluid ounce of water has a weight of roughly one ounce avoirdupois (actually 1 fluid ounce of water weighs 1.043176 ounces). One pint (16 fluid ounces) of water has a weight of roughly 1 pound (16 ounces) avoirdupois (actually one pint of water weighs 1.043176 pounds). One fluid ounce equals two tablespoons.

6.7 Troy Units

It is customary to measure precious metals (gold, silver, platinum, palladium, and rhodium) in *troy* units. In this system, a troy ounce is 31.1034768 grams. Twelve (12) troy ounces makes one troy pound (373.24172 grams). One troy ounce equals 480 grains (exactly); one troy pound is 5760 grains.

Gold dealers and commodity markets like the Chicago Board of Trade will quote gold and silver prices in dollars per ounce. It's understood by everyone involved that "ounce" in this case refers to *troy* ounces. Precious metals are *always* measured in troy units — *never* in avoirdupois units.

6.8 Units as an Error-Checking Technique

Checking units can be used as an important error-checking technique called *dimensional analysis*. If you derive an equation and find that the units don't work out properly, then you can be certain you made a mistake somewhere. If the units are correct, it doesn't necessarily mean your derivation is correct (since you could be off by a factor of 2, for example), but it does give you some confidence that you at least haven't made a units error. So checking units doesn't tell you for certain whether or not you've made a mistake, but it does help.

Here are some basic principles to keep in mind when working with units:

1. Units on both sides of an equation must match.
2. When adding or subtracting two quantities, they must have the same units.
3. Quantities that appear in exponents must be dimensionless.
4. The argument for functions like sin, cos, tan, $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, log, and exp must be dimensionless.
5. When checking units, radians and steradians can be considered dimensionless.
6. When checking complicated units, it may be useful to break down all derived units into base units (e.g. replace newtons with kg m s^{-2}).

Sometimes it's not clear whether or not the units match on both sides of the equation, for example when both sides involve derived SI units. In that case, it may be useful to break all the derived units down in terms of base SI units (m, kg, s, A, K, mol, cd). Table 20-2 in Appendix 20 shows each of the derived SI units broken down in terms of base SI units.

6.9 Unit Conversions

It is very common to have to work with quantities that are given in units other than the units you'd like to work with. Converting from one set of units to another involves a straightforward, virtually foolproof technique that's very simple to double-check. We'll illustrate the method here with some examples.

Appendix 25 gives a number of important conversion factors. More conversion factors are available from sources such as the *CRC Handbook of Chemistry and Physics*.

1. Write down the unit conversion factor as a ratio, and fill in the units in the numerator and denominator so that the units cancel out as needed.

2. Now fill in the numbers so that the numerator and denominator contain the same length, time, etc. (This is because you want each factor to be a multiplication by 1, so that you don't change the quantity—only its units.)

Simple Conversions

A simple unit conversion involves only one conversion factor. The method for doing the conversion is best illustrated with an example.

Example. Convert 7 feet to inches.
Solution. First write down the unit conversion factor as a ratio, filling in the units as needed:

$$(7\text{ ft}) \times \frac{\text{in}}{\text{ft}} \tag{6.8}$$

Notice that the units of feet cancel out, leaving units of inches. The next step is to fill in numbers so that the same length is in the numerator and denominator:

$$(7\text{ ft}) \times \frac{12\text{ in}}{1\text{ ft}} \tag{6.9}$$

Now do the arithmetic:

$$(7\text{ ft}) \times \frac{12\text{ in}}{1\text{ ft}} = 84\text{ inches.} \tag{6.10}$$

More Complex Conversions

More complex conversions may involve more than one conversion factor. You'll need to think about what conversion factors you know, then put together a chain of them to get to the units you want.

Example. Convert 60 miles per hour to feet per second.
Solution. First, write down a chain of conversion factor ratios, filling in units so that they cancel out correctly:

$$60\ \frac{\text{mile}}{\text{hr}} \times \frac{\text{ft}}{\text{mile}} \times \frac{\text{hr}}{\text{sec}} \tag{6.11}$$

Units cancel out to leave ft/sec. Now fill in the numbers, putting the same length in the numerator and denominator in the first factor, and the same time in the numerator and denominator in the second factor:

$$60\ \frac{\text{mile}}{\text{hr}} \times \frac{5280\text{ ft}}{1\text{ mile}} \times \frac{1\text{ hr}}{3600\text{ sec}} \tag{6.12}$$

Finally, do the arithmetic:

$$60\ \frac{\text{mile}}{\text{hr}} \times \frac{5280\text{ ft}}{1\text{ mile}} \times \frac{1\text{ hr}}{3600\text{ sec}} = 88\ \frac{\text{ft}}{\text{sec}} \tag{6.13}$$

Example. Convert 250,000 furlongs per fortnight to meters per second.

Solution. We don't know how to convert furlongs per fortnight directly to meters per second, so we'll have to come up with a chain of conversion factors to do the conversion. We *do* know how to convert: furlongs to miles, miles to kilometers, kilometers to meters, fortnights to weeks, weeks to days, days to hours, hours to minutes, and minutes to seconds. So we start by writing conversion factor ratios, putting units where they need to be so that the result will have the desired target units (m/s):

$$250{,}000\ \frac{\text{furlong}}{\text{fortnight}} \times \frac{\text{mile}}{\text{furlong}} \times \frac{\text{km}}{\text{mile}} \times \frac{\text{m}}{\text{km}} \times \frac{\text{fortnight}}{\text{week}} \times \frac{\text{week}}{\text{day}} \times \frac{\text{day}}{\text{hr}} \times \frac{\text{hr}}{\text{min}} \times \frac{\text{min}}{\text{sec}}$$

If you check the units here, you'll see that almost everything cancels out; the only units left are m/s, which is what we want to convert to. Now fill in the numbers: we want to put either the same length or the same time in both the numerator and denominator:

$$\begin{aligned} &250{,}000\ \frac{\text{furlong}}{\text{fortnight}} \times \frac{1\ \text{mile}}{8\ \text{furlongs}} \times \frac{1.609344\ \text{km}}{1\ \text{mile}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{fortnight}}{2\ \text{weeks}} \times \frac{1\ \text{week}}{7\ \text{days}} \times \frac{1\ \text{day}}{24\ \text{hr}} \times \frac{1\ \text{hr}}{60\ \text{min}} \times \frac{1\ \text{min}}{60\ \text{sec}} \\ &= 41.58\ \text{m/s} \end{aligned}$$

Conversions Involving Powers

Occasionally we need to do something like convert an area or volume when we know only the length conversion factor.

Example. Convert 2000 cubic feet to gallons.

Solution. Let's think about what conversion factors we know. We know the conversion factor between gallons and cubic inches. We don't know the conversion factor between cubic feet and cubic inches, but we can convert between feet and inches. The conversion factors will look like this:

$$2000\ \text{ft}^3 \times \left(\frac{\text{in}}{\text{ft}}\right)^3 \times \frac{\text{gal}}{\text{in}^3} \tag{6.14}$$

With these units, the whole expression reduces to units of gallons. Now fill in the same length in the numerator and denominator of the first factor, and the same volume in the numerator and denominator of the second factor:

$$2000\ \text{ft}^3 \times \left(\frac{12\ \text{in}}{1\ \text{ft}}\right)^3 \times \frac{1\ \text{gal}}{231\ \text{in}^3} \tag{6.15}$$

Now do the arithmetic:

$$2000\ \text{ft}^3 \times \left(\frac{12\ \text{in}}{1\ \text{ft}}\right)^3 \times \frac{1\ \text{gal}}{231\ \text{in}^3} = 14{,}961\ \text{gallons} \tag{6.16}$$

6.10 Currency Units

Money has units that can be treated like any other units, using the same techniques we've just seen. Two things are unique about units of currency:

- Each country has its own currency units. Examples are United States dollars ($), British pounds sterling (£), European euros (€), and Japanese yen (¥).
- The conversion factors from one country's currency to another's is a function of time, and even varies minute to minute during the day. These conversion factors are called *exchange rates*, and may be found, for example, on the Internet at

  `http://www.xe.com/currencyconverter/`

Example. You're shopping in Reykjavík, Iceland, and see an Icelandic wool scarf you'd like to buy. The price tag says 6990 kr. What is the price in U.S. dollars?

Solution. The unit of currency in Iceland is the Icelandic króna (kr). Looking up the exchange rate on the Internet, you find it is currently \$1 = 119.050 kr. Then

$$6990 \text{ kr.} \times \frac{\$1.00}{119.050 \text{ kr.}} = \$58.71 \tag{6.17}$$

A quick note on notation: for United States money, the dollar sign (\$) goes *before* the number, but the cents sign (¢) goes *after* the number. For example: \$10, \$12.45, 79¢, 49¢. For very large numbers, the name is written *after* the numbers: \$10 million, \$1.4 trillion.

For euros, the euro symbol (€) may be written either before or after the number: €10, or 10 €.

6.11 Some Units Puzzles

Lead vs. Feathers?

Question. Which weighs more: a pound of lead or a pound of feathers?

Solution. They're the same. Lead may be much *denser* than feathers, but a pound of lead is the same as a pound of feathers, 453.5924 grams.

Feathers vs. Gold

Question. Which weighs more: a pound of feathers or a pound of gold?

Solution. In this case they're *not* the same. Feathers are measured in pounds avoirdupois (the "usual" pounds unit we use every day in the United States). But gold, like all precious metals, is measured in *troy* units. A pound (avoirdupois) of feathers is 453.5924 grams, but a pound of gold would be one *troy* pound, which is 373.2417 grams. A pound of feathers weighs more.

Milk vs. Strawberries

Question. Which is bigger, a quart of milk or a quart of strawberries?

Solution. Once again, they're not the same units. Milk is measured in U.S. liquid quarts; one liquid quart is 0.9463529 liters. But strawberries and other produce are measured in *dry quarts*; one dry quart is 1.101221 liters. A quart of strawberries is bigger.

Soda Pop vs. Gold?

Question. Which weighs more, 16 ounces of soda pop, or a pound of solid gold?

Solution. The soda pop weighs more. 16 ounces of soda pop is 16 FLUID ounces — drinks are sold by volume, not by weight. In the avoirdupois system, 1 fluid ounce of water weighs roughly 1 ounce, so 16 fluid ounces (1 pint) of soda pop (which is mostly water) weighs about 16 ounces avoirdupois, or 1 pound avoirdupois = 459.53 grams. For a more careful analysis, 16 fluid ounces = 473.18 cm^3. The density of

Coca-Cola is 1.042 g/cm^3, so the mass of soda pop is mass = density × volume = (1.042)(473.18) = 493.05 grams.

Gold, like all precious metals (Au, Ag, Pt, Pd, Rh) is always measured in troy units. One troy pound of gold = 12 troy ounces = 373.24 grams, which is significantly less than the mass of soda pop (493.05 grams).

6.12 A Peculiar Example

The author recently received a rather enigmatic comment on a social media post:

A víte, co stojí vejce na Václavském náměstí?

This is a bit of a puzzle. Where do we start to understand this? First, the Internet is very helpful in identifying the language: it's Czech. It translates as:

Do you know what eggs cost on Wenceslas Square?

Wenceslas Square is a marketplace in Prague, confirming that the language is Czech. We've made some headway.

Now to solve the problem. According to the Internet, eggs in the Czech Republic cost between US 29¢ and 92¢ per pound. Let's say 92¢ per pound, because we're going to buy the best quality grade A eggs (large). Some further research on the Internet shows that a large sized chicken egg has an average weight of about 2 ounces. This should be enough information to determine the price *per egg* in the Czech Republic:

$$\frac{92\text{ cents}}{\text{lb}} \times \frac{1\text{ lb}}{16\text{ oz}} \times \frac{2\text{ oz}}{1\text{ egg}} = 11.5\text{ cents/egg}$$

Eggs are normally sold by the dozen, so a dozen eggs is

$$\frac{11.5\text{ cents}}{1\text{ egg}} \times \frac{12\text{ eggs}}{1\text{ dozen}} = 138\text{ cents/dozen} = \text{US }\$1.38/\text{dozen}$$

Now we refer to the Internet to find the exchange rate. The Czech unit of currency is the Czech *koruna* (Kč), with 100 haléře per koruna. At the time of this writing, the exchange rate was 1.00 US Dollar = 22.792804 Czech koruny (so a koruna is roughly 1 US nickel). Doing the currency conversion from dollars to koruny:

$$\frac{\$1.38}{\text{dozen}} \times \frac{22.792804\text{ koruny}}{1.00\text{ dollar}} = 31.45\text{ koruny/dozen}$$

Apparently hellers have been withdrawn from circulation, so we would round to the nearest koruna: 31 Kč per dozen.

Finally, we call on the Internet once again to translate back into Czech. In English:

On Wenceslas Square, eggs cost 31 koruny per dozen.

Translating into Czech gives us the proper social media response:

Na Václavském náměstí stojí vejce 31 korun za tucet.

6.13 Odds and Ends

We'll end this chapter with a few miscellaneous notes about SI units:

- In a few special cases, we customarily drop the ending vowel of a prefix when combining with a unit that begins with a vowel: it's *megohm* (*not* "megaohm"); *kilohm* (*not* "kiloohm"); and *hectare* (*not* "hectoare"). In all other cases, keep both vowels (e.g. *microohm*, *kiloare*, etc.). There's no particular reason for this—it's just customary.

- In pharmacology (on bottles of vitamins or prescription medicine, for example), it is usual to indicate micrograms with "mcg" rather than "μg". While this is technically incorrect, it is done to avoid misreading the units. Using "mc" for "micro" is not done outside pharmacology, and you should not use it in physics. Always use μ for "micro".

- Sometimes in electronics work the SI prefix symbol may be used in place of the decimal point. For example, 24.9 MΩ may be written "24M9". This saves space on electronic diagrams and when printing values on electronic components, and also avoids problems with the decimal point being nearly invisible when the print is tiny. This is unofficial use, and is only encountered in electronics.

- One sometimes encounters older metric units of length called the *micron* (μ, now properly called the *micrometer*, 10^{-6} meter) and the *millimicron* (mμ, now properly called the *nanometer*, 10^{-9} meter). The micron and millimicron are now obsolete.

- At one time there was a metric prefix *myria-* (my) that meant 10^4. This prefix is obsolete and is no longer used.

- In computer work, the SI prefixes are often used with units of bytes, but may refer to powers of 2 that are near the SI values. For example, the term "1 kB" may mean 1000 bytes, or it may mean $2^{10} = 1024$ bytes. Similarly, a 100 GB hard drive may have a capacity of 100,000,000,000 bytes, or it may mean $100 \times 2^{30} = 107{,}374{,}182{,}400$ bytes. To help resolve these ambiguities, a set of *binary prefixes* has been introduced (Table 20-4 of Appendix 20). These prefixes have not yet entirely caught on in the computing industry, though.

6.14 Problems

1. An Olympic-size swimming pool is 50 meters long, 25 meters wide, and 2 meters deep. How much water is this in: (a) cubic meters; (b) cubic feet; (c) gallons; (d) teaspoons?

2. If the distance from New York to Los Angeles is 2445 miles, then convert this distance to: (a) kilometers; (b) inches; (c) barleycorns. (A *barleycorn* is a unit equal to $\frac{1}{3}$ inch.)

3. According to a recent story in the *Washington Post*, a package containing a pair of riding boots was mailed from South Charleston, West Virginia in 1979, and finally arrived at its destination in Laurel, Maryland in 2020 — 41 years later. The total distance traveled was 363 miles. (a) What was the average speed of the package? (Speed = total distance divided by time.) Give your answer in units of inches per minute. (b) A snail can travel with a speed of about 0.003 miles per hour. Convert this to inches per minute. How does the speed of the package in part (a) compare to the speed of a snail?

4. If Peter Piper picks a peck of pickled peppers per picosecond, then how many bushels of pickled peppers does Peter Piper pick per hour?

5. In the United States, paper thickness is often measured in units of "pounds" (e.g. "20-pound paper"). This refers to the weight, in pounds, of 1 ream (500 sheets) of 17 in $\times$ 22 in paper, which is exactly four times the size of U.S. standard "letter-sized" paper.

   In Europe, paper thickness is instead measured in grams per square meter (g/m^2, often marked "gsm") — the mass, in grams, of one square meter of paper.

   (a) Derive formulæ for converting US "pounds" paper thickness to and from units of g/m^2. (b) Using your formulæ, convert 20-pound paper to g/m^2. (You may be able to check your answer by finding a package of 20-pound paper; it may have the European gsm rating on it.)

6. United States dollar bills (properly called *Federal Reserve notes*) are printed on a special type of linen paper, whose exact composition is a closely guarded secret. Federal Reserve notes have a mass of 1 gram, and measure 6.14×2.61 inches. (a) Compute the area density of a Federal Reserve note in g/m^2. (b) Using the results of the previous problem, convert your result from part (a) to paper "pounds."

Chapter 7

Problem-Solving Strategies

Much of this course will focus on developing your ability to solve physics problems. If you enjoy solving puzzles, you'll find solving physics problems is similar in many ways. Here we'll look at a few general tips on how to approach solving problems.

- At the beginning of a problem stated in SI units, immediately convert the units of all the quantities you're given to base SI units. In other words, convert all lengths to meters, all masses to kilograms, all times to seconds, etc.: all quantities should be in un-prefixed SI units, except for masses in kilograms. When you do this, you're guaranteed that the final result will also be in base SI units, and this will minimize your problems with units. As you gain more experience in problem solving, you'll sometimes see shortcuts that let you get around this suggestion, but for now converting all units to base SI units is the safest approach.

- Similarly, if the problem is stated in CGS units immediately convert all given quantities to base CGS units (lengths in centimeters, masses in grams, and times in seconds). If the problem is stated in British engineering units, immediately convert all given quantities to base units (lengths in feet, masses in slugs, and times in seconds).

- Look at the information you're given, and what you're being asked to find. Then think about what equations you know that might let you get from what you're given to what you're trying to find.

- Be sure you understand under what conditions each equation is valid. For example, we'll shortly see a set of equations that are derived by assuming constant acceleration. It would be inappropriate to use those equations for a mass on a spring, since the acceleration of a mass under a spring force is *not* constant. For each equation you're using, you should be clear what each variable represents, and under what conditions the equation is valid.

- As a general rule, it's best to derive an algebraic expression for the solution to a problem first, then substitute numbers to compute a numerical answer as the very last step. This approach has a number of advantages: it allows you to check units in your algebraic expression, helps minimize roundoff error, and allows you to easily repeat the calculation for different numbers if needed.

- If you've derived an algebraic equation, *check the units* of your answer. Make sure your equation has the correct units, and doesn't do something like add quantities with different units.

- If you've derived an algebraic equation, you can check that it has the proper behavior for extreme values of the variables. For example, does the answer make sense if time $t \to \infty$? If the equation contains an angle, does it reduce to a sensible answer when the angle is $0°$ or $90°$?

- Check your answer for reasonableness—don't just write down whatever your calculator says. For example, suppose you're computing the speed of a pendulum bob in the laboratory, and find the answer is 14,000 miles per hour. That doesn't seem reasonable, so you should go back and check your work.
- You can avoid rounding errors by carrying as many significant digits as possible throughout your calculations; don't round off until you get to the final result.
- Write down a reasonable number of significant digits in the final answer—don't write down all the digits in your calculator's display. Nor should you round too much and use too few significant digits. There are rules for determining the correct number of significant digits, but for most problems in this course, 3 or 4 significant digits will be about right.
- Don't forget to put the correct units on the final answer! You will have points deducted for forgetting to do this.
- The best way to get good at problem solving (and to prepare for exams for this course) is *practice*—practice working as many problems as you have time for. Working physics problems is a skill much like learning to play a sport or musical instrument. You can't learn by watching someone else do it—you can only learn it by doing it yourself.

Chapter 8

Measurement

8.1 Measurement

8.2 Calculations with Measurements

Chapter 9

Vectors

We will next want to extend our knowledge of kinematics from one dimension to two and three dimensions. However, the equations will be expressed in the mathematical language of *vectors*, so we'll need to examine the mathematics of vectors first.

9.1 Introduction

Some quantities we measure in physics have only a *magnitude*; such quantities are called *scalars*. Examples of scalars are mass and temperature. Other quantities have both a magnitude and a *direction*; such quantities are called *vectors*. Examples of vectors are velocity, acceleration, and electric field.

You can represent a vector graphically by drawing an *arrow*. The direction of the arrow indicates the direction of the vector, while the length of the arrow represents the magnitude of the vector on some chosen scale. By convention, we write vector names in boldface type in typeset text (e.g. **A**); when writing vectors by hand, it is customary to draw a small arrow over the name (e.g. $\vec{A}$).

Besides drawing a vector in the plane of the page, occasionally you may want to draw a vector diagram in which you want to indicate a vector pointing directly into or out of the page. You can do this using these symbols:

Symbol	Meaning
$\longrightarrow$	Vector *in plane of* page
$\otimes$	Vector *into* page
$\odot$	Vector *out of* page

The symbol $\otimes$ is supposed to look like the tail feathers of an arrow flying away from you, while the symbol $\odot$ is supposed to resemble the head of an arrow flying directly toward you. Of course, if you use these two symbols, you can only indicate the *direction* of the vector, not its magnitude—but this is often all that's needed.

It is possible to do arithmetic on vectors: for example, you can add or subtract two vectors, or multiply a vector by a scalar. These operations may be done either graphically or algebraically. Both methods will be described here.

9.2 Vector Arithmetic: Graphical Methods

Vector arithmetic can be done graphically, by drawing the vectors as arrows on graph paper, and measuring the results with a ruler and protractor. The advantage of the graphical methods are that they give a good intuitive picture of what's going on to help you visualize what you're trying to do. The disadvantages are that the graphical methods can be time-consuming, and not very accurate.

In practice, the graphical methods are usually used to make a quick sketch, to help organize and clarify your thinking, so you can be clear that you're doing things correctly. The algebraic methods are then used for the actual calculations.

When drawing vectors, you are free to move the vector around the page however you want, as long as you don't change the direction or magnitude.

Addition

We'll begin with addition. There are two methods available to add two vectors together: the first is called the *parallelogram method*. In this method, you draw the two vectors to be added with their tail end points at the same point. This figure forms half a parallelogram; draw two additional lines to complete the parallelogram. Now draw a vector from the tail endpoint across the diagonal of the parallelogram. This diagonal vector is the sum of the two original vectors (Fig. 9.1(a)).

The second graphical method of vector addition is called the *triangle method*. In this method, you first draw one vector, then draw the second so that its tail is at the head of the first vector. To find the sum of the two vectors, draw a vector from the tail of the first vector to the head of the second (Fig. 9.1(b)).

The triangle method can be extended to add any number of vectors together. Just draw the vectors one by one, with the tail of each vector at the head of the previous one. The sum of all the vectors is then found by drawing a vector from the tail of the first vector in the chain to the head of the last one (Fig. 9.1(c)). This is called the *polygon method*.

Subtraction

To subtract two vectors graphically, draw the two vectors so that their tail endpoints are at the same point. To draw the difference vector, draw a vector from the head of the subtrahend vector to the head of the minuend vector (Fig. 9.1(d)).

Scalar Multiplication

Multiplying a vector by a scalar will change the length of the vector. Multiplying by a scalar greater than 1 (in absolute value) will lengthen the vector; multiplying by a scalar less than 1 in absolute value will shrink the vector. If the scalar is positive, the product vector will have the same direction as the original; if the scalar is negative, the product vector will be opposite the direction of the original (Fig. 9.1(e)).

9.3 Vector Arithmetic: Algebraic Methods

Although the graphical methods just described give a good intuitive picture of the mathematical operations, they can be a bit tedious to draw. A much more convenient and accurate alternative is the set of *algebraic* methods, which involve working with numbers instead of graphs. Before we can do that, though, we need to find a way to quantify a vector—to change it from a graph of an arrow to a set of numbers we can work with.

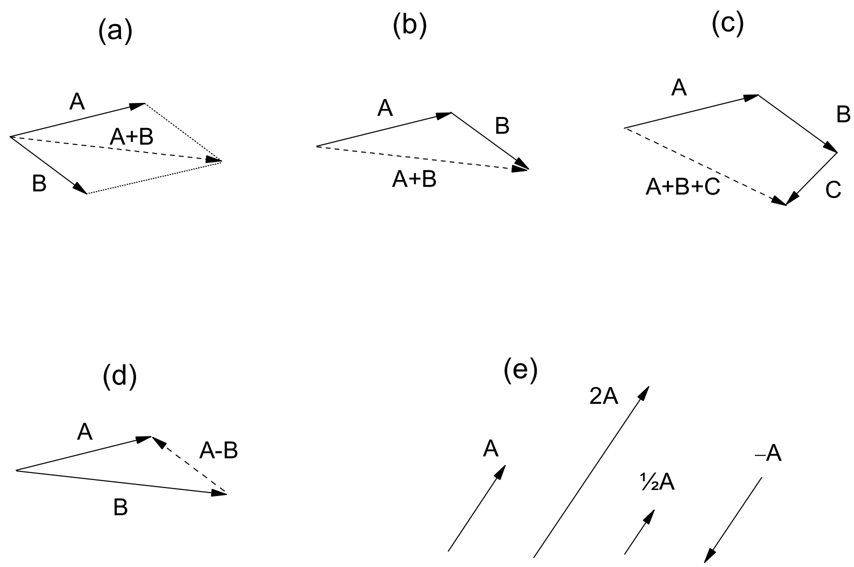


Figure 9.1: Graphical methods for vector arithmetic. (a) Addition of vectors **A** and **B** using the parallelogram method. (b) Addition of the same vectors **A** and **B** using the triangle method. (c) Addition of vectors **A**, **B**, and **C** using a generalization of the triangle method called the *polygon method*. The sum vector points from the tail of the first vector to the head of the last. (d) Vector subtraction: $\mathbf{A} - \mathbf{B}$ points from the head of **B** to the head of **A**. (e) Multiplication of a vector **A** by various scalars. Multiplying by a scalar greater than 1 makes the vector longer; multiplying by a scalar less than 1 makes it shorter. The resulting vector will be in the same direction as **A** unless the scalar is negative, in which case the result will point opposite the direction of **A**.

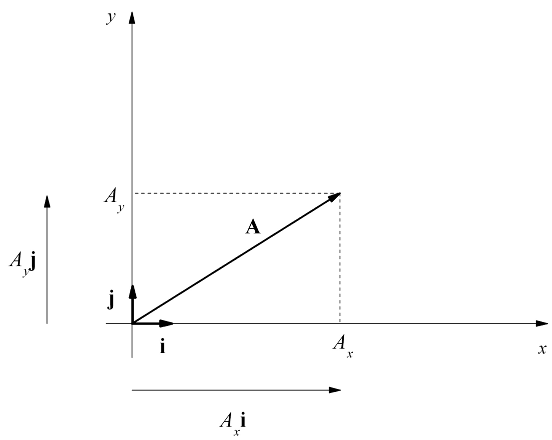


Figure 9.2: Cartesian components of a vector.

Rectangular Form

One idea would be to keep track of the coordinates of the head and tail of the vector. But remember that we are free to move a vector around whereever we want, as long as the direction and magnitude remain unchanged. So let's choose to always put the tail of the vector at the origin—that way, we only have to keep track of the head of the vector, and we cut our work in half. A vector can then be completely specified by just giving the coordinates of its head.

There's a little bit of a different way of writing this, though. We begin by defining two *unit vectors* (vectors with magnitude 1): **i** is a unit vector in the x direction, and **j** is a unit vector in the y direction. (In three dimensions, we add a third unit vector **k** in the z direction.)

Referring to Fig. 9.2, let A_x be the projection of vector **A** onto the x-axis, and let A_y by its projection onto the y-axis. Then, recalling the rules for the multiplication of a vector by a scalar, $A_x\mathbf{i}$ is a vector pointing in the x-direction, and whose length is equal to the projection A_x. Similarly, $A_y\mathbf{j}$ is a vector pointing in the y-direction, and whose length is equal to the projection A_y. Then by the parallelogram rule for adding two vectors, vector **A** is the sum of vectors $A_x\mathbf{i}$ and $A_y\mathbf{j}$ (Fig. 9.2). This means we can write a vector **A** as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}, \tag{9.1}$$

or, if we're working in three dimensions,

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}. \tag{9.2}$$

Eq. (9.1) or (9.2) is called the *rectangular* or *cartesian*[1] form of vector **A**.

[1] The name *cartesian* is from *Cartesius*, the Latin form of the name of the French mathematician René Descartes, the founder of analytic geometry.

Magnitude

The *magnitude* of a vector is a measure of its total "length." It is indicated with absolute value signs around the vector ($|\mathbf{A}|$ in type, or $|\vec{A}|$ in handwriting), or more simply by just writing the name of the vector in regular type (A; no boldface or arrow). In terms of rectangular components, the magnitude of a vector is simply given by the Pythagorean theorem:

$$|\mathbf{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}. \tag{9.3}$$

Example. The magnitude of vector $\mathbf{A} = 2\mathbf{i} + 5\mathbf{j}$ is

$$|\mathbf{A}| = A = \sqrt{2^2 + 5^2} = \sqrt{29} = \boxed{5.3852}$$

Polar Form

Instead of giving the x and y coordinates of the head of the vector, an alternative form is to give the magnitude and direction of the vector. This is called the *polar* form of a vector, and is indicated by the notation

$$\mathbf{A} = A\angle\theta, \tag{9.4}$$

where A is the magnitude of the vector, and θ is the direction, measured counterclockwise from the $+x$ axis.

By convention, in polar form, we always take the magnitude of a vector as positive. If the magnitude comes out negative (as the result of a calculation, for example), then we can make it positive by changing its sign and adding 180° to the direction.

Converting between the rectangular and polar forms of a vector is fairly straightforward. To convert from polar to rectangular form, we use the definitions of the sine and cosine to get $\sin\theta = \text{opp}/\text{hyp} = A_y/A$, and $\cos\theta = \text{adj}/\text{hyp} = A_x/A$. Therefore to convert from polar to rectangular form, we use

$$A_x = A\cos\theta \tag{9.5}$$

$$A_y = A\sin\theta \tag{9.6}$$

To go the other way (rectangular to polar form), we just invert these equations to solve for A and θ. To solve for A, take the sum of the squares of both equations and add; to solve for θ, divide the A_y equation by the A_x equation. The results are

$$A = \sqrt{A_x^2 + A_y^2} \tag{9.7}$$

$$\tan\theta = \frac{A_y}{A_x} \tag{9.8}$$

To find θ, you must take the arctangent of the right-hand side if Eq. (9.8). But be careful: to get the angle in the correct quadrant, you first compute the right-hand side of Eq. (9.8), then use the arctangent (`TAN`$^{-1}$) function on your calculator. If $A_x > 0$, then the calculator shows θ. But if $A_x < 0$, you must remember to add 180° (π rad) to the calculator's answer to get θ in the correct quadrant.

It is also possible to write three-dimensional vectors in polar form, but this requires a magnitude and *two* angles. We won't have any need to write three-dimensional vectors in polar form for this course.

Example: Polar to rectangular. Convert the vector $\mathbf{A} = 7\angle 40^\circ$ from polar form to rectangular form:

$$A_x = A\cos\theta = 7\cos 40^\circ = 5.3623 \tag{9.9}$$
$$A_y = A\sin\theta = 7\sin 40^\circ = 4.4995 \tag{9.10}$$

so the rectangular form is $\mathbf{A} = 5.3623\,\mathbf{i} + 4.4995\,\mathbf{j}$

Example: Rectangular to polar. Convert the vector $\mathbf{B} = -4\,\mathbf{i} + 8\,\mathbf{j}$ from rectangular form to polar form:

$$|\mathbf{B}| = \sqrt{B_x^2 + B_y^2} = \sqrt{(-4)^2 + 8^2} = \sqrt{16+64} = \sqrt{80} = 8.9443 \tag{9.11}$$
$$\tan\theta = \frac{B_y}{B_x} = \frac{8}{-4} = -2 \qquad \Rightarrow \qquad \theta = 116.565^\circ \tag{9.12}$$

so the polar form is $8.9443\angle 116.565^\circ$.

Notice that to find θ, we take the inverse tangent of -2 and the calculator returns -63.435°. But because the denominator (-4) is *negative*, we add 180° to the calculator's answer: $-63.435^\circ + 180^\circ = 116.565^\circ$. If the denominator had been *positive*, we would *not* have added this 180°. For example, the rectangular vector $\mathbf{C} = 4\,\mathbf{i} - 8\,\mathbf{j}$ would be $8.9443\angle - 64.435^\circ$ in polar form.

Vector Equality

In order for two vectors to be *equal*, they must have the same magnitude and point in the same direction. This means that each of their components must be equal. For example, if $\mathbf{A} = \mathbf{B}$, then all of the following must be true:

$$A_x = B_x \tag{9.13}$$
$$A_y = B_y \tag{9.14}$$
$$A_z = B_z \tag{9.15}$$

Addition

Now we're ready to describe the algebraic method for the addition of two vectors. First, both vectors *must be in rectangular (cartesian) form*—you *cannot* add vectors in polar form. If you're given two vector in polar form and must add them, you must first convert them to rectangular form using Eq. (9.5-9.6).

Once the vectors are in rectangular form, you simply add the two vectors component by component: the x-component of the sum is the sum of the x components, etc.:

$$\begin{array}{c|l} \mathbf{A} & = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \\ +\,\mathbf{B} & = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k} \\ \hline \mathbf{A}+\mathbf{B} & = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k} \end{array}$$

Subtraction

Just as with addition, vectors must be in rectangular (cartesian) form before they can be subtracted. Vector subtraction is similar to vector addition: you simply subtract the two vectors component by component:

$$\begin{array}{c|l} \mathbf{A} & = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \\ -\,\mathbf{B} & = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k} \\ \hline \mathbf{A}-\mathbf{B} & = (A_x - B_x)\mathbf{i} + (A_y - B_y)\mathbf{j} + (A_z - B_z)\mathbf{k} \end{array}$$

Scalar Multiplication

Multiplication of a vector by a scalar may be done in either rectangular or polar form. In rectangular form, you multiply each component of the vector by the scalar. For example, given the vector **A** and scalar c:

$$c\mathbf{A} = c(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \tag{9.16}$$
$$= cA_x\mathbf{i} + cA_y\mathbf{j} + cA_z\mathbf{k} \tag{9.17}$$

It's even simpler in polar form: if the vector $\mathbf{A} = A\angle\theta$, then

$$c\mathbf{A} = (cA)\angle\theta. \tag{9.18}$$

It's conventional to keep the vector magnitude positive, so if $cA < 0$, you should change the sign of the magnitude cA, then add 180° (π radians) to the angle θ.

Example: Addition. Add the vectors $\mathbf{A} = 6\,\mathbf{i} - 9\,\mathbf{j}$ and $\mathbf{B} = 2\,\mathbf{i} + 12\,\mathbf{j}$:

$$\begin{array}{cl} \mathbf{A} & = 6\,\mathbf{i} - 9\,\mathbf{j} \\ +\,\mathbf{B} & = 2\,\mathbf{i} + 12\,\mathbf{j} \\ \hline \mathbf{A} + \mathbf{B} & = 8\,\mathbf{i} + 3\,\mathbf{j} \end{array}$$

Example: Subtraction. Subtract the vectors $\mathbf{A} = 6\,\mathbf{i} - 9\,\mathbf{j}$ and $\mathbf{B} = 2\,\mathbf{i} + 12\,\mathbf{j}$:

$$\begin{array}{cl} \mathbf{A} & = 6\,\mathbf{i} - 9\,\mathbf{j} \\ -\,\mathbf{B} & = 2\,\mathbf{i} + 12\,\mathbf{j} \\ \hline \mathbf{A} - \mathbf{B} & = 4\,\mathbf{i} - 21\,\mathbf{j} \end{array}$$

Example: Scalar multiplication. Multiply the vector $\mathbf{A} = 6\,\mathbf{i} - 9\,\mathbf{j}$ by 5:

$$5 \times (6\,\mathbf{i} - 9\,\mathbf{j}) = 30\,\mathbf{i} - 45\,\mathbf{j}$$

9.4 The Zero Vector

The *zero vector* is the vector $\mathbf{0} = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}$. It has zero magnitude, and its direction is undefined. The zero vector is *not* the same thing as the scalar 0: $\mathbf{0} \neq 0$. One is a vector, and the other is a scalar.

9.5 Other Vector Operations

Other mathematical operations with vectors are possible. For example, is it possible to *add* a vector and a scalar together? The answer is: sort of. You get something similar to a *quaternion*, which is a hypercomplex number of the form $a + bi + cj + dk$ (where $i^2 = j^2 = k^2 = -1$). Quaternions are sometimes used in aeronautical and astronautical engineering to describe the rotation of one coordinate system with respect to another.

What about multiplying a vector by another vector? Yes, this is possible. In fact, there are *three* different kinds of multiplication that can be used to multiply two vectors together, as described in the next chapter.

How about division—can you divide by a vector? No; division by a vector is not defined. A vector may be a dividend, but not a divisor. But you can divide a vector by a *scalar* by simply multiplying by the reciprocal of the scalar:

$$\frac{\mathbf{A}}{c} = \frac{1}{c}(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \tag{9.19}$$

$$= \frac{A_x}{c}\mathbf{i} + \frac{A_y}{c}\mathbf{j} + \frac{A_z}{c}\mathbf{k}. \tag{9.20}$$

Chapter 10

Motion

10.1 Kinematics in One Dimension

Coordinate Systems

Constant Velocity

$$\text{distance} = \text{speed} \times \text{time} \tag{10.1}$$

$$x(t) = vt + x_0 \tag{10.2}$$

Constant Acceleration

$$x(t) = \tfrac{1}{2}at^2 + v_0 t + x_0 \tag{10.3}$$
$$v(t) = at + v_0 \tag{10.4}$$
$$v^2 = v_0^2 + 2a(x - x_0) \tag{10.5}$$

10.2 Acceleration Due to Gravity; Helmert's Equation

An important special case of constant acceleration is the case of bodies falling near the surface of the Earth under the influence of gravity. All falling bodies fall downward with the same constant acceleration, called the *acceleration due to gravity* g, which is equal to

$$g = G\frac{M_\oplus}{R_\oplus^2} = 9.8\ \mathrm{m/s^2} = 32\ \mathrm{ft/s}, \tag{10.6}$$

to two significant digits, where $M_\oplus$ is the Earth's mass and $R_\oplus$ is its radius.

What if we want to use a more exact value for g? You might be tempted to use a value found in some reference books: $g = 9.80665$ m/s^2, but that would actually be wrong. This value is just a standard value used for the definitions of some units (for example, in the conversion between pounds-force and newtons). You should never use this value in a physics formula that contains g as the acceleration due to gravity—it's only used when doing certain unit conversions.

The acceleration due to gravity g at the surface of the Earth varies over the surface of the Earth for a number of reasons:

1. As you get closer to the equator, the Earth's rotation rate gets larger, resulting in a greater centrifugal force that counteracts gravity. This has the effect of reducing g closer to the equator.
2. Also, the Earth has an equatorial bulge due to its rotation, so that you're farther from the center of the Earth near the equator. This also has the effect of reducing g closer to the equator.
3. There is also an elevation effect: the higher you are in elevation, the smaller g is.

These effects can be approximately accounted for using an equation called *Helmert's equation*. According to Helmert's equation, the acceleration due to gravity is given by

$$\boxed{g = 9.80616 - 0.025928\cos 2\phi + (6.9\times 10^{-5})\cos^2 2\phi - (3.086\times 10^{-6})H \quad \mathrm{m/s^2},} \tag{10.7}$$

where ϕ is the latitude and H is the elevation (in meters) above sea level. For example, for Largo, Maryland, the latitude ϕ is $38.^\circ898$ and the elevation H is about 174 feet (53.0 meters). Substituting these values into Helmert's equation, we find g at Largo is about 9.80052 m/s^2. In other cities around the world, the value ranges from 9.779 m/s^2 (Mexico City) to 9.819 m/s^2 (Helsinki). For most problems we just use an average value of 9.8 m/s^2. (You should *never* round this to 10 m/s^2 unless you're doing a very rough order-of-magnitude estimation.)

10.3 Kinematics in Two and Three Dimensions

Projectile Motion

The Monkey-Hunter Problem

Chapter 11

Force

Chapter 12

Work and Energy

Chapter 13

Gravitation

Chapter 14

Waves and Sound

Chapter 15

Light

Chapter 16

Reflection and Refraction

Chapter 17

Color and Physical Optics

Part III

Remote Sensing

Chapter 18

Remote Sensing Basics

Chapter 19

Electromagnetic Radiation

Chapter 20

Remote Sensing Platforms

Part IV

Ethics

Chapter 21

Professional Ethics

Physics, like all sciences, requires its practitioners to be scrupulously honest. Dishonesty makes good science impossible.

Below are a few thoughts on good scientific practice.[1]

1. *One must* never *fabricate data or falsify results in any way.* Let the data stand on its own. Do not fabricate data, or manipulate it just to make the results come out the way you want.

2. *Do not plagiarize.* It is normal, and expected, that you will often cite work done by others in your own work. Always give them due credit for their work. It's better to give someone too much credit than not enough.

3. *Keep your personal political views out of the science.* Although we live in an age when politics overshadows almost everything, it is crucial to keep politics away from science. The result of scientific research must be based on evidence — not on what you might *want* to be true, or what you think *should* be true for personal or political reasons.

4. *Be objective.* It is crucial to scientific research that scientists be completely be objective in their work. Don't let your work be biased in favor of what results you *want* to get. Try as hard as you can to disprove your own research and find things wrong with it. Expect your colleagues to do the same.

5. *Recognize that there is only one "truth" to Nature.* There is objectively one set of natural laws that governs Nature. There is no "my truth" or "your truth" — only *the* truth.

6. *Do not take criticism personally.* Other scientists will review and criticize your work and try to find fault with it. That's exactly as it should be. Be prepared to defend your work from invalid criticism, but also be willing to accept valid criticism. Again, it's important to be objective.

7. *Make every effort to understand other points of view.* Scientists often have differences of opinion about things. Ultimately, objective evidence will sort out whose opinion is consistent with Nature. You should recognize that people have valid opinions that differ from yours.

8. *Do not participate in personal attacks.* Calling other scientists names like "truth denier" is very bad behavior indeed. Name-calling is for children. Have respect for other scientists and their views. For example, years ago, there was a debate in cosmology over which model of the Universe was correct:

[1] The opinions expressed in this chapter are entirely those of the author, and do not necessarily represent those of Prince George's Community College, the Department of Natural Sciences, or its faculty or staff.

the Big Bang model, or the Steady State model. Eventually it came down to only a single scientist still advocating for the Steady State model. Nobody mocked him or called him a "Big Bang denier." He had his views, and everyone in the scientific community respected that.

9. *Stand up for your views if you believe they're correct, but also be willing to admit when you're wrong.* You'll earn a lot of respect if you can recognize when the evidence simply does not support your hypothesis, and are willing to admit you were wrong. (And you should thank people who point out your errors.)

10. *Skepticism is healthy for science.* Scientists *should* be skeptical about scientific theories. We should constantly be trying to find flaws in accepted theories. To do otherwise is to accept dogma, rather than pursuing scientific investigation. Calling someone a "denier" for being skeptical of a theory is not science.

11. *Above all, always be scrupulously honest.* Always be completely honest, even when it may not be to your advantage. Being honest is more important than personal gain.

It's important to recognize something very important about scientific research: *no experiment or observation will ever prove that a theory is "correct."* Experiments may prove a theory to be wrong, if its predictions do not match observations. But if a theory's predictions *do* match observations, then all we've shown is that the theory is "consistent with observation." A theory may make predictions that match observations, yet still be wrong, because future observations may be inconsistent with the theory and prove the theory wrong.

In science, a "theory" is simply a best guess. We're constantly making guesses about how Nature works, and testing those guesses against observation. When a theory is inconsistent with observation, we know the theory is wrong and must be modified or replaced. It follows from this that the popular saying, "the science is settled" is false. That would imply that scientific investigation is up for some kind of one-time "vote" by the scientific community, and that once the vote is taken, that subject may never be investigated or questioned ever again. That's just *not* how science works. Science is *never* "settled" — it is always subject to questioning, skepticism, re-examination, and re-investigation. Otherwise, it is not science.

A rough analogy might be putting together a jigsaw puzzle. As you put each piece in place, you start to get an idea of what the big picture (the whole puzzle) looks like. But at any time, you might have some pieces out of place, and those pieces need to be identified and removed. It may even be that you have large parts of the puzzle completely wrong, and large sections must be removed and replaced. But at any given instant, the current state of the puzzle is your best guess about how the whole thing goes together.

Another important principle in scientific research is known as Occam's razor, which states that the simplest explanation for a phenomenon is usually the correct one.[2] For example, if you see an unusual object in the sky, what might it be? An aircraft, or an extraterrestrial spacecraft carrying aliens from another world? It might be either one, but good scientific practice says that we assume the simplest, most mundane explanation is most likely — a terrestrial aircraft.

We may consider it a corollary to Occam's razor that extraordinary claims require extraordinary evidence. We should probably not believe that the object is an alien spacecraft unless there is an extraordinary amount of supporting evidence, and no other explanation is possible. Similarly with reported sightings of "Big Foot" or the Loch Ness Monster — they would be extraordinary discoveries if true, so we should demand an overwhelming amount of evidence before accepting them as fact.

A good lesson in the (im)proper conducting of scientific research may be learned from an incident that happened in 1989, when two chemists believed that they had produced nuclear fusion using an electrochemical setup in the laboratory — a process called "cold fusion." If true, it would be an extraordinary discovery — one normally produces nuclear fusion in a particle accelerator, not in laboratory glassware. If they had

[2]Named for the fourteenth-century Franciscan friar William of Ockham, it was originally stated as, "Numquam ponenda est pluralitas sine necessitate," or "Plurality must never be posited without necessity."

followed good scientific practice, the two researchers would have first tried to repeat their experiment over and over and tried to get others to repeat it. After being certain that their experiment was repeatable, they should have submitted their findings as a paper to be published in a peer-reviewed scientific journal so that others could critique and try to duplicate and verify their findings, and perhaps offer alternative explanations for the results.

Instead, the two researchers bypassed the peer-review process and went immediately to the news media to announce their findings, jumping to conclusions and claiming they had discovered "cold fusion." It created quite a sensation in the scientific community for a time, but ultimately other groups were not able to reproduce their findings. The researchers' scientific reputations were ruined, and all because they jumped to conclusions and bypassed the usual peer-review process.

Finally, Nobel laureate Richard Feynman once said, "The first principle is that you must not fool yourself — and you are the easiest person to fool." The truth may not be what you'd like it to be, or what would be best for you, or what your preconceived philosophy tells you that it is. Unless you recognize how easily you can be fooled, you will be. [6]

Appendices

Appendix 1

Hints for Selected Problems

Chapter 2

Appendix 2

Solutions to Selected Problems

Chapter 2

Appendix 3

Greek Alphabet

Table 3-1. The Greek alphabet.

Letter	Name
$\mathrm{A}\ \alpha$	Alpha
$\mathrm{B}\ \beta$	Beta
$\Gamma\ \gamma$	Gamma
$\Delta\ \delta$	Delta
$\mathrm{E}\ \varepsilon$	Epsilon
$\mathrm{Z}\ \zeta$	Zeta
$\mathrm{H}\ \eta$	Eta
$\Theta\ \theta$	Theta
$\mathrm{I}\ \iota$	Iota
$\mathrm{K}\ \kappa$	Kappa
$\Lambda\ \lambda$	Lambda
$\mathrm{M}\ \mu$	Mu
$\mathrm{N}\ \nu$	Nu
$\Xi\ \xi$	Xi
O o	Omicron
$\Pi\ \pi$	Pi
$\mathrm{P}\ \rho$	Rho
$\Sigma\ \sigma$	Sigma
$\mathrm{T}\ \tau$	Tau
$\Upsilon\ \upsilon$	Upsilon
$\Phi\ \varphi$	Phi
$\mathrm{X}\ \chi$	Chi
$\Psi\ \psi$	Psi
$\Omega\ \omega$	Omega

(Alternate forms: $ϐ = \beta$, $\epsilon = \varepsilon$, $\vartheta = \theta$, $\varkappa = \kappa$, $\varpi = \pi$, $\varrho = \rho$, $\varsigma = \sigma$, Y$= \Upsilon$, $\phi = \varphi$.)

Appendix 4

Arithmetic Tables

Addition Table

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Multiplication Table

×	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	12	23	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	13	24	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625

Appendix 5

Prime Numbers

Table 5-1. Prime numbers less than 1000.

2	3	5	7	11	13	17	19
23	29	31	37	41	43	47	53
59	61	67	71	73	79	83	89
97	101	103	107	109	113	127	131
137	139	149	151	157	163	167	173
179	181	191	193	197	199	211	223
227	229	233	239	241	251	257	263
269	271	277	281	283	293	307	311
313	317	331	337	347	349	353	359
367	373	379	383	389	397	401	409
419	421	431	433	439	443	449	457
461	463	467	479	487	491	499	503
509	521	523	541	547	557	563	569
571	577	587	593	599	601	607	613
617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719
727	733	739	743	751	757	761	769
773	787	797	809	811	821	823	827
829	839	853	857	859	863	877	881
883	887	907	911	919	929	937	941
947	953	967	971	977	983	991	997

Appendix 6

Prime Factorizatons

Table 6-1. Prime factorizations of integers to 100.

1	—	26	2×13	51	3×17	76	$2^2 \times 19$
2	2	27	3^3	52	$2^2 \times 13$	77	7×11
3	3	28	$2^2 \times 7$	53	53	78	$2 \times 3 \times 13$
4	2^2	29	29	54	2×3^3	79	79
5	5	30	$2 \times 3 \times 5$	55	5×11	80	$2^4 \times 5$
6	2×3	31	31	56	$2^3 \times 7$	81	3^4
7	7	32	2^5	57	3×19	82	2×41
8	2^3	33	3×11	58	2×29	83	83
9	3^2	34	2×17	59	59	84	$2^2 \times 3 \times 7$
10	2×5	35	5×7	60	$2^2 \times 3 \times 5$	85	5×17
11	11	36	$2^2 \times 3^2$	61	61	86	2×43
12	$2^2 \times 3$	37	37	62	2×31	87	3×29
13	13	38	2×19	63	$3^2 \times 7$	88	$2^3 \times 11$
14	2×7	39	3×13	64	2^6	89	89
15	3×5	40	$2^3 \times 5$	65	5×13	90	$2 \times 3^2 \times 5$
16	2^4	41	41	66	$2 \times 3 \times 11$	91	7×13
17	17	42	$2 \times 3 \times 7$	67	67	92	$2^2 \times 23$
18	2×3^2	43	43	68	$2^2 \times 17$	93	3×31
19	19	44	$2^2 \times 11$	69	3×23	94	2×47
20	$2^2 \times 5$	45	$3^2 \times 5$	70	$2 \times 5 \times 7$	95	5×19
21	3×7	46	2×23	71	71	96	$2^5 \times 3$
22	2×11	47	47	72	$2^3 \times 3^2$	97	97
23	23	48	$2^4 \times 3$	73	73	98	2×7^2
24	$2^3 \times 3$	49	7^2	74	2×37	99	$3^2 \times 11$
25	5^2	50	2×5^2	75	3×5^2	100	$2^2 \times 5^2$

Appendix 7

Julian Day Conversions

The following algorithms are taken from *Astronomical Algorithms* by Jean Meeus [5]. Here $\mathrm{INT}(x)$ means taking the integer part of x.

Calendar Date to Julian Day

Let Y be the year, M the month number (1 to 12), and D the day of month (including fraction of a day). Then:

- If $M = 1$ or $M = 2$, then replace Y by $Y - 1$ and M by $M + 12$. (If $M > 2$, then leave Y and M unchanged.)
- Compute A and B:

  $$A = \mathrm{INT}\left(\frac{Y}{100}\right) \qquad B = 2 - A + \mathrm{INT}\left(\frac{A}{4}\right)$$

  (If using the Old-Style Julian calendar, instead take $B = 0$.)
- The Julian Day is then

  $$\mathrm{JD} = \mathrm{INT}[365.25(Y + 4716)] + \mathrm{INT}[30.6001(M + 1)] + D + B + 1524.5 \tag{7.1}$$

Julian Day to Calendar Date

- Add 0.5 to the Julian Day. Let Z be the integer part, and F the fractional (decimal) part of the result.
- If $Z < 2299161$, take $A = Z$. Otherwise, find

  $$\alpha = \mathrm{INT}\left(\frac{Z - 1867216.25}{36524.25}\right)$$
  $$A = Z + 1 + \alpha - \mathrm{INT}\left(\frac{\alpha}{4}\right)$$

- Then calculate

$$
\begin{aligned}
B &= A + 1524 \\
C &= \mathrm{INT}\left(\frac{B - 122.1}{365.25}\right) \\
D &= \mathrm{INT}(365.25C) \\
E &= \mathrm{INT}\left(\frac{B - D}{30.6001}\right)
\end{aligned}
$$

- The day of the month (with decimals, if any) is then

$$B - D - \mathrm{INT}(30.6001D) + F$$

- The month number m is

$$
\begin{array}{ll}
E - 1 & \text{if } E < 14 \\
E - 13 & \text{if } E = 14 \text{ or } E = 15
\end{array}
$$

- The year is

$$
\begin{array}{ll}
C - 4716 & \text{if } m > 2 \\
C - 4715 & \text{if } m = 1 \text{ or } m = 2
\end{array}
$$

Appendix 8

Derivation of the Quadratic Formula

The general quadratic equation is

$$ax^2 + bx + c = 0 \tag{8.1}$$

To solve for x, we first divide both sides by a:

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Now subtract c/a from both sides:

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \tag{8.2}$$

Next, add $\left(\frac{b}{2a}\right)^2$ to both sides so we can complete the square:

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \tag{8.3}$$

Simplifying,

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a}\frac{4a}{4a} + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2} \tag{8.4}$$

Take the square root of both sides:

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \tag{8.5}$$

Finally, we subtract $b/2a$ from both sides:

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \tag{8.6}$$

or

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{8.7}$$

which is the quadratic formula.

Appendix 9

Binomial Coefficients (Pascal's Triangle)

${}_nC_r = \binom{n}{r}$: Values of n in left column; values of r in top row.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1														
2	1	2	1													
3	1	3	3	1												
4	1	4	6	4	1											
5	1	5	10	10	5	1										
6	1	6	15	20	15	6	1									
7	1	7	21	35	35	21	7	1								
8	1	8	28	56	70	56	28	8	1							
9	1	9	36	84	126	126	84	36	9	1						
10	1	10	45	120	210	252	210	120	45	10	1					
11	1	11	55	165	330	462	462	330	165	55	11	1				
12	1	12	66	220	495	792	924	792	495	220	66	12	1			
13	1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1		
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1	
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105	15	1

Appendix 10

Table of Quarter Squares

N	Q.S.	N	Q.S.	N	Q.S.	N	Q.S.	N	Q.S.	N	Q.S.
0	0	50	625	100	2500	150	5625	200	10000	250	15625
1	0	51	650	101	2550	151	5700	201	10100	251	15750
2	1	52	676	102	2601	152	5776	202	10201	252	15876
3	2	53	702	103	2652	153	5852	203	10302	253	16002
4	4	54	729	104	2704	154	5929	204	10404	254	16129
5	6	55	756	105	2756	155	6006	205	10506	255	16256
6	9	56	784	106	2809	156	6084	206	10609	256	16384
7	12	57	812	107	2862	157	6162	207	10712	257	16512
8	16	58	841	108	2916	158	6241	208	10816	258	16641
9	20	59	870	109	2970	159	6320	209	10920	259	16770
10	25	60	900	110	3025	160	6400	210	11025	260	16900
11	30	61	930	111	3080	161	6480	211	11130	261	17030
12	36	62	961	112	3136	162	6561	212	11236	262	17161
13	42	63	992	113	3192	163	6642	213	11342	263	17292
14	49	64	1024	114	3249	164	6724	214	11449	264	17424
15	56	65	1056	115	3306	165	6806	215	11556	265	17556
16	64	66	1089	116	3364	166	6889	216	11664	266	17689
17	72	67	1122	117	3422	167	6972	217	11772	267	17822
18	81	68	1156	118	3481	168	7056	218	11881	268	17956
19	90	69	1190	119	3540	169	7140	219	11990	269	18090
20	100	70	1225	120	3600	170	7225	220	12100	270	18225
21	110	71	1260	121	3660	171	7310	221	12210	271	18360
22	121	72	1296	122	3721	172	7396	222	12321	272	18496
23	132	73	1332	123	3782	173	7482	223	12432	273	18632
24	144	74	1369	124	3844	174	7569	224	12544	274	18769
25	156	75	1406	125	3906	175	7656	225	12656	275	18906
26	169	76	1444	126	3969	176	7744	226	12769	276	19044
27	182	77	1482	127	4032	177	7832	227	12882	277	19182
28	196	78	1521	128	4096	178	7921	228	12996	278	19321
29	210	79	1560	129	4160	179	8010	229	13110	279	19460
30	225	80	1600	130	4225	180	8100	230	13225	280	19600
31	240	81	1640	131	4290	181	8190	231	13340	281	19740
32	256	82	1681	132	4356	182	8281	232	13456	282	19881
33	272	83	1722	133	4422	183	8372	233	13572	283	20022
34	289	84	1764	134	4489	184	8464	234	13689	284	20164
35	306	85	1806	135	4556	185	8556	235	13806	285	20306
36	324	86	1849	136	4624	186	8649	236	13924	286	20449
37	342	87	1892	137	4692	187	8742	237	14042	287	20592
38	361	88	1936	138	4761	188	8836	238	14161	288	20736
39	380	89	1980	139	4830	189	8930	239	14280	289	20880
40	400	90	2025	140	4900	190	9025	240	14400	290	21025
41	420	91	2070	141	4970	191	9120	241	14520	291	21170
42	441	92	2116	142	5041	192	9216	242	14641	292	21316
43	462	93	2162	143	5112	193	9312	243	14762	293	21462
44	484	94	2209	144	5184	194	9409	244	14884	294	21609
45	506	95	2256	145	5256	195	9506	245	15006	295	21756
46	529	96	2304	146	5329	196	9604	246	15129	296	21904
47	552	97	2352	147	5402	197	9702	247	15252	297	22052
48	576	98	2401	148	5476	198	9801	248	15376	298	22201
49	600	99	2450	149	5550	199	9900	249	15500	299	22350

Appendix 11

Trigonometry

Pythagorean Formulæ

$$\sin^2\theta + \cos^2\theta \equiv 1$$
$$\sec^2\theta \equiv 1 + \tan^2\theta$$
$$\csc^2\theta \equiv 1 + \cot^2\theta$$

Angle Addition Formulæ

$$\sin(\alpha \pm \beta) \equiv \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) \equiv \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) \equiv \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Double-Angle Formulæ

$$\sin 2\theta \equiv 2\sin\theta\cos\theta \equiv \frac{2\tan\theta}{1+\tan^2\theta}$$
$$\cos 2\theta \equiv \cos^2\theta - \sin^2\theta \equiv 1 - 2\sin^2\theta \equiv 2\cos^2\theta - 1 \equiv \frac{1-\tan^2\theta}{1+\tan^2\theta}$$
$$\tan 2\theta \equiv \frac{2\tan\theta}{1-\tan^2\theta}$$

Triple-Angle Formulæ

$$\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta$$
$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$$
$$\tan 3\theta \equiv \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$
$$\cot 3\theta \equiv \frac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}$$

Quadruple-Angle Formulæ

$$\sin 4\theta \equiv 4\cos^3\theta \sin\theta - 4\cos\theta \sin^3\theta$$
$$\cos 4\theta \equiv \cos^4\theta - 6\cos^2\theta \sin^2\theta + \sin^4\theta$$
$$\tan 4\theta \equiv \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$$
$$\cot 4\theta \equiv \frac{\cot^4\theta - 6\cot^2\theta + 1}{4\cot^3\theta - 4\cot\theta}$$

Half-Angle Formulæ

$$\sin\frac{\theta}{2} \equiv \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\frac{\theta}{2} \equiv \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\frac{\theta}{2} \equiv \frac{\sin\theta}{1+\cos\theta} \equiv \frac{1-\cos\theta}{\sin\theta}$$

Products to Sums

$$\sin\alpha\ \cos\beta \equiv \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$
$$\cos\alpha\ \sin\beta \equiv \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$
$$\cos\alpha\ \cos\beta \equiv \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$
$$\sin\alpha\ \sin\beta \equiv -\frac{1}{2}\left[\cos(\alpha+\beta) - \cos(\alpha-\beta)\right]$$
$$\tan\alpha\ \tan\beta \equiv \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{\cos(\alpha-\beta) + \cos(\alpha+\beta)}$$

Sums to Products

$$\sin\alpha + \sin\beta \equiv 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\sin\alpha - \sin\beta \equiv 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$
$$\cos\alpha + \cos\beta \equiv 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha - \cos\beta \equiv -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$
$$\tan\alpha + \tan\beta \equiv \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$$
$$\tan\alpha - \tan\beta \equiv \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$$

Power Reduction Formulæ

$$\sin^2\theta \equiv \tfrac{1}{2}\left(1-\cos 2\theta\right)$$

$$\cos^2\theta \equiv \tfrac{1}{2}\left(1+\cos 2\theta\right)$$

$$\tan^2\theta \equiv \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

Other Formulæ

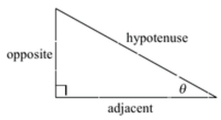
$$\tan\theta \equiv \cot\theta - 2\cot 2\theta$$

Trig Cheat Sheet

Definition of the Trig Functions

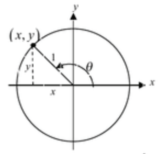
Right triangle definition
For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0^\circ < \theta < 90^\circ$.



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Unit circle definition
For this definition θ is any angle.



$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$
$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$
$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Facts and Properties

Domain
The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle
$\cos\theta$, θ can be any angle
$\tan\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \ldots$
$\csc\theta$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$
$\sec\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \ldots$
$\cot\theta$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

Range
The range is all possible values to get out of the function.

$-1 \le \sin\theta \le 1 \qquad \csc\theta \ge 1$ and $\csc\theta \le -1$
$-1 \le \cos\theta \le 1 \qquad \sec\theta \ge 1$ and $\sec\theta \le -1$
$-\infty < \tan\theta < \infty \qquad -\infty < \cot\theta < \infty$

Period
The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$
$$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$
$$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$
$$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$
$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$
$$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$
$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$
$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$
$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$
$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas
If n is an integer.

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$
$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$
$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$
$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Degrees to Radians Formulas
If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas

$$\sin^2\theta = \frac{1}{2}(1 - \cos(2\theta))$$
$$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$$
$$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$
$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$
$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]$$
$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$
$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$
$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$
$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$
$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$
$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

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Exact values of trigonometric functions at 3° intervals. (Ref. [3])

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0^\circ = 0\pi$	0	1	0
$3^\circ = \frac{\pi}{60}$	$\frac{1}{16}\left[\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{5}-1\right)-2\left(\sqrt{3}-1\right)\sqrt{5+\sqrt{5}}\right]$	$\frac{1}{16}\left[2\left(\sqrt{3}+1\right)\sqrt{5+\sqrt{5}}+\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{5}-1\right)\right]$	$\frac{1}{4}\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}-1\right)\left(\sqrt{10+2\sqrt{5}}-\sqrt{5}-1\right)$
$6^\circ = \frac{\pi}{30}$	$\frac{1}{8}\left(\sqrt{30-6\sqrt{5}}-\sqrt{5}-1\right)$	$\frac{1}{8}\left(\sqrt{15}+\sqrt{3}+\sqrt{10-2\sqrt{5}}\right)$	$\frac{1}{2}\left(\sqrt{10-2\sqrt{5}}-\sqrt{15}+\sqrt{3}\right)$
$9^\circ = \frac{\pi}{20}$	$\frac{1}{8}\left(\sqrt{10}+\sqrt{2}-2\sqrt{5-\sqrt{5}}\right)$	$\frac{1}{8}\left(\sqrt{10}+\sqrt{2}+2\sqrt{5-\sqrt{5}}\right)$	$\sqrt{5}+1-\sqrt{5+2\sqrt{5}}$
$12^\circ = \frac{\pi}{15}$	$\frac{1}{8}\left(\sqrt{10+2\sqrt{5}}-\sqrt{15}+\sqrt{3}\right)$	$\frac{1}{8}\left(\sqrt{30+6\sqrt{5}}+\sqrt{5}-1\right)$	$\frac{1}{2}\left(3\sqrt{3}-\sqrt{15}-\sqrt{50-22\sqrt{5}}\right)$
$15^\circ = \frac{\pi}{12}$	$\frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$	$\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$	$2-\sqrt{3}$
$18^\circ = \frac{\pi}{10}$	$\frac{1}{4}\left(\sqrt{5}-1\right)$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{1}{5}\sqrt{25-10\sqrt{5}}$
$21^\circ = \frac{7\pi}{60}$	$\frac{1}{16}\left[2\left(\sqrt{3}+1\right)\sqrt{5-\sqrt{5}}-\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{5}+1\right)\right]$	$\frac{1}{16}\left[\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{5}+1\right)+2\left(\sqrt{3}-1\right)\sqrt{5-\sqrt{5}}\right]$	$\frac{1}{4}\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}+1\right)\left(\sqrt{10-2\sqrt{5}}-\sqrt{5}+1\right)$
$24^\circ = \frac{2\pi}{15}$	$\frac{1}{8}\left(\sqrt{15}+\sqrt{3}-\sqrt{10-2\sqrt{5}}\right)$	$\frac{1}{8}\left(\sqrt{30-6\sqrt{5}}+\sqrt{5}+1\right)$	$\frac{1}{2}\left(\sqrt{50+22\sqrt{5}}-3\sqrt{3}-\sqrt{15}\right)$
$27^\circ = \frac{3\pi}{20}$	$\frac{1}{8}\left(2\sqrt{5+\sqrt{5}}-\sqrt{10}+\sqrt{2}\right)$	$\frac{1}{8}\left(2\sqrt{5+\sqrt{5}}+\sqrt{10}-\sqrt{2}\right)$	$\sqrt{5}-1-\sqrt{5-2\sqrt{5}}$
$30^\circ = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$
$33^\circ = \frac{11\pi}{60}$	$\frac{1}{16}\left[\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{5}-1\right)+2\left(\sqrt{3}-1\right)\sqrt{5+\sqrt{5}}\right]$	$\frac{1}{16}\left[2\left(\sqrt{3}+1\right)\sqrt{5+\sqrt{5}}-\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{5}-1\right)\right]$	$\frac{1}{4}\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}-1\right)\left(\sqrt{10+2\sqrt{5}}+\sqrt{5}+1\right)$
$36^\circ = \frac{\pi}{5}$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\frac{1}{4}\left(\sqrt{5}+1\right)$	$\sqrt{5-2\sqrt{5}}$
$39^\circ = \frac{13\pi}{60}$	$\frac{1}{16}\left[\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{5}+1\right)-2\left(\sqrt{3}-1\right)\sqrt{5-\sqrt{5}}\right]$	$\frac{1}{16}\left[2\left(\sqrt{3}+1\right)\sqrt{5-\sqrt{5}}+\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{5}+1\right)\right]$	$\frac{1}{4}\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}-1\right)\left(\sqrt{10-2\sqrt{5}}-\sqrt{5}+1\right)$
$42^\circ = \frac{7\pi}{30}$	$\frac{1}{8}\left(\sqrt{30+6\sqrt{5}}-\sqrt{5}+1\right)$	$\frac{1}{8}\left(\sqrt{10+2\sqrt{5}}+\sqrt{15}-\sqrt{3}\right)$	$\frac{1}{2}\left(\sqrt{15}+\sqrt{3}-\sqrt{10+2\sqrt{5}}\right)$
$45^\circ = \frac{\pi}{4}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$	1
$48^\circ = \frac{4\pi}{15}$	$\frac{1}{8}\left(\sqrt{10+2\sqrt{5}}+\sqrt{15}-\sqrt{3}\right)$	$\frac{1}{8}\left(\sqrt{30+6\sqrt{5}}-\sqrt{5}+1\right)$	$\frac{1}{2}\left(3\sqrt{3}-\sqrt{15}+\sqrt{50-22\sqrt{5}}\right)$
$51^\circ = \frac{17\pi}{60}$	$\frac{1}{16}\left[2\left(\sqrt{3}+1\right)\sqrt{5-\sqrt{5}}+\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{5}+1\right)\right]$	$\frac{1}{16}\left[\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{5}+1\right)-2\left(\sqrt{3}-1\right)\sqrt{5-\sqrt{5}}\right]$	$\frac{1}{4}\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}+1\right)\left(\sqrt{10-2\sqrt{5}}+\sqrt{5}-1\right)$
$54^\circ = \frac{3\pi}{10}$	$\frac{1}{4}\left(\sqrt{5}+1\right)$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\frac{1}{5}\sqrt{25+10\sqrt{5}}$
$57^\circ = \frac{19\pi}{60}$	$\frac{1}{16}\left[2\left(\sqrt{3}+1\right)\sqrt{5+\sqrt{5}}-\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{5}-1\right)\right]$	$\frac{1}{16}\left[\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{5}-1\right)+2\left(\sqrt{3}-1\right)\sqrt{5+\sqrt{5}}\right]$	$\frac{1}{4}\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}+1\right)\left(\sqrt{10+2\sqrt{5}}-\sqrt{5}-1\right)$
$60^\circ = \frac{\pi}{3}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}$	$\sqrt{3}$
$63^\circ = \frac{7\pi}{20}$	$\frac{1}{8}\left(2\sqrt{5+\sqrt{5}}+\sqrt{10}-\sqrt{2}\right)$	$\frac{1}{8}\left(2\sqrt{5+\sqrt{5}}-\sqrt{10}+\sqrt{2}\right)$	$\sqrt{5}-1+\sqrt{5-2\sqrt{5}}$
$66^\circ = \frac{11\pi}{30}$	$\frac{1}{8}\left(\sqrt{30-6\sqrt{5}}+\sqrt{5}+1\right)$	$\frac{1}{8}\left(\sqrt{15}+\sqrt{3}-\sqrt{10-2\sqrt{5}}\right)$	$\frac{1}{2}\left(\sqrt{10-2\sqrt{5}}+\sqrt{15}-\sqrt{3}\right)$
$69^\circ = \frac{23}{60}\pi$	$\frac{1}{16}\left[\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{5}+1\right)+2\left(\sqrt{3}-1\right)\sqrt{5-\sqrt{5}}\right]$	$\frac{1}{16}\left[2\left(\sqrt{3}+1\right)\sqrt{5-\sqrt{5}}-\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{5}+1\right)\right]$	$\frac{1}{4}\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}-1\right)\left(\sqrt{10-2\sqrt{5}}+\sqrt{5}-1\right)$
$72^\circ = \frac{2\pi}{5}$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{1}{4}\left(\sqrt{5}-1\right)$	$\sqrt{5+2\sqrt{5}}$
$75^\circ = \frac{5\pi}{12}$	$\frac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$	$\frac{1}{4}\left(\sqrt{6}-\sqrt{2}\right)$	$2+\sqrt{3}$
$78^\circ = \frac{13\pi}{30}$	$\frac{1}{8}\left(\sqrt{30+6\sqrt{5}}+\sqrt{5}-1\right)$	$\frac{1}{8}\left(\sqrt{10+2\sqrt{5}}-\sqrt{15}+\sqrt{3}\right)$	$\frac{1}{2}\left(\sqrt{15}+\sqrt{3}+\sqrt{10+2\sqrt{5}}\right)$
$81^\circ = \frac{19\pi}{20}$	$\frac{1}{8}\left(\sqrt{10}+\sqrt{2}+2\sqrt{5-\sqrt{5}}\right)$	$\frac{1}{8}\left(\sqrt{10}+\sqrt{2}-2\sqrt{5-\sqrt{5}}\right)$	$\sqrt{5}+1+\sqrt{5+2\sqrt{5}}$
$84^\circ = \frac{7\pi}{15}$	$\frac{1}{8}\left(\sqrt{15}+\sqrt{3}+\sqrt{10-2\sqrt{5}}\right)$	$\frac{1}{8}\left(\sqrt{30-6\sqrt{5}}-\sqrt{5}-1\right)$	$\frac{1}{2}\left(\sqrt{50+22\sqrt{5}}+3\sqrt{3}+\sqrt{15}\right)$
$87^\circ = \frac{29\pi}{60}$	$\frac{1}{16}\left[2\left(\sqrt{3}+1\right)\sqrt{5+\sqrt{5}}+\left(\sqrt{6}-\sqrt{2}\right)\left(\sqrt{5}-1\right)\right]$	$\frac{1}{16}\left[\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{5}-1\right)-2\left(\sqrt{3}-1\right)\sqrt{5+\sqrt{5}}\right]$	$\frac{1}{4}\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}+1\right)\left(\sqrt{10+2\sqrt{5}}+\sqrt{5}+1\right)$
$90^\circ = \frac{\pi}{2}$	1	0	—

θ	$\sec\theta$	$\csc\theta$	$\cot\theta$
$0^\circ = 0\pi$	1	—	—
$3^\circ = \frac{\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(\sqrt{5+2\sqrt{5}}-2+\sqrt{3}\right)$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(2+\sqrt{3}+\sqrt{5+2\sqrt{5}}\right)$	$\frac{1}{4}\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}+1\right)\left(\sqrt{10+2\sqrt{5}}+\sqrt{5}+1\right)$
$6^\circ = \frac{\pi}{30}$	$\sqrt{3}-\sqrt{5-2\sqrt{5}}$	$\sqrt{15+6\sqrt{5}}+\sqrt{5}+2$	$\frac{1}{2}\left(\sqrt{50+22\sqrt{5}}+3\sqrt{3}+\sqrt{15}\right)$
$9^\circ = \frac{\pi}{20}$	$\frac{1}{2}\left(3\sqrt{2}+\sqrt{10}-2\sqrt{5+\sqrt{5}}\right)$	$\frac{1}{2}\left(3\sqrt{2}+\sqrt{10}+2\sqrt{5}+\sqrt{5}\right)$	$\sqrt{5}+1+\sqrt{5+2\sqrt{5}}$
$12^\circ = \frac{\pi}{15}$	$\sqrt{15-6\sqrt{5}}-\sqrt{5}+2$	$\sqrt{5+2\sqrt{5}}+\sqrt{3}$	$\frac{1}{2}\left(\sqrt{15}+\sqrt{3}+\sqrt{10+2\sqrt{5}}\right)$
$15^\circ = \frac{\pi}{12}$	$\sqrt{6}-\sqrt{2}$	$\sqrt{6}+\sqrt{2}$	$2+\sqrt{3}$
$18^\circ = \frac{\pi}{10}$	$\frac{1}{5}\sqrt{50-10\sqrt{5}}$	$\sqrt{5}+1$	$\sqrt{5+2\sqrt{5}}$
$21^\circ = \frac{7\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(2+\sqrt{3}-\sqrt{5-2\sqrt{5}}\right)$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(\sqrt{5-2\sqrt{5}}+2-\sqrt{3}\right)$	$\frac{1}{4}\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}-1\right)\left(\sqrt{10-2\sqrt{5}}+\sqrt{5}-1\right)$
$24^\circ = \frac{2\pi}{15}$	$\sqrt{15+6\sqrt{5}}-\sqrt{5}-2$	$\sqrt{3}+\sqrt{5-2\sqrt{5}}$	$\frac{1}{2}\left(\sqrt{10-2\sqrt{5}}+\sqrt{15}-\sqrt{3}\right)$
$27^\circ = \frac{3\pi}{20}$	$\frac{1}{2}\left(2\sqrt{5-\sqrt{5}}-3\sqrt{2}+\sqrt{10}\right)$	$\frac{1}{2}\left(2\sqrt{5-\sqrt{5}}+3\sqrt{2}-\sqrt{10}\right)$	$\sqrt{5}-1+\sqrt{5-2\sqrt{5}}$
$30^\circ = \frac{\pi}{6}$	$\frac{2}{3}\sqrt{3}$	2	$\sqrt{3}$
$33^\circ = \frac{11\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(\sqrt{5+2\sqrt{5}}+2-\sqrt{3}\right)$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(2+\sqrt{3}-\sqrt{5+2\sqrt{5}}\right)$	$\frac{1}{4}\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}+1\right)\left(\sqrt{10+2\sqrt{5}}-\sqrt{5}-1\right)$
$36^\circ = \frac{\pi}{5}$	$\sqrt{5}-1$	$\frac{1}{5}\sqrt{50+10\sqrt{5}}$	$\frac{1}{5}\sqrt{25+10\sqrt{5}}$
$39^\circ = \frac{13\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(\sqrt{5-2\sqrt{5}}-2+\sqrt{3}\right)$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(2+\sqrt{3}+\sqrt{5-2\sqrt{5}}\right)$	$\frac{1}{4}\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}+1\right)\left(\sqrt{10-2\sqrt{5}}+\sqrt{5}-1\right)$
$42^\circ = \frac{7\pi}{30}$	$\sqrt{5+2\sqrt{5}}-\sqrt{3}$	$\sqrt{15-6\sqrt{5}}+\sqrt{5}-2$	$\frac{1}{2}\left(3\sqrt{3}-\sqrt{15}+\sqrt{50-22\sqrt{5}}\right)$
$45^\circ = \frac{\pi}{4}$	$\sqrt{2}$	$\sqrt{2}$	1
$48^\circ = \frac{4\pi}{15}$	$\sqrt{15-6\sqrt{5}}+\sqrt{5}-2$	$\sqrt{5+2\sqrt{5}}-\sqrt{3}$	$\frac{1}{2}\left(\sqrt{15}+\sqrt{3}-\sqrt{10+2\sqrt{5}}\right)$
$51^\circ = \frac{17\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(2+\sqrt{3}+\sqrt{5-2\sqrt{5}}\right)$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(\sqrt{5-2\sqrt{5}}-2+\sqrt{3}\right)$	$\frac{1}{4}\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}-1\right)\left(\sqrt{10-2\sqrt{5}}-\sqrt{5}+1\right)$
$54^\circ = \frac{3\pi}{10}$	$\frac{1}{5}\sqrt{50+10\sqrt{5}}$	$\sqrt{5}-1$	$\sqrt{5-2\sqrt{5}}$
$57^\circ = \frac{19\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(2+\sqrt{3}-\sqrt{5+2\sqrt{5}}\right)$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(\sqrt{5+2\sqrt{5}}+2-\sqrt{3}\right)$	$\frac{1}{4}\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}-1\right)\left(\sqrt{10+2\sqrt{5}}+\sqrt{5}+1\right)$
$60^\circ = \frac{\pi}{3}$	2	$\frac{2}{3}\sqrt{3}$	$\frac{1}{3}\sqrt{3}$
$63^\circ = \frac{7\pi}{20}$	$\frac{1}{2}\left(2\sqrt{5-\sqrt{5}}+3\sqrt{2}-\sqrt{10}\right)$	$\frac{1}{2}\left(2\sqrt{5-\sqrt{5}}-3\sqrt{2}+\sqrt{10}\right)$	$\sqrt{5}-1-\sqrt{5-2\sqrt{5}}$
$66^\circ = \frac{11\pi}{30}$	$\sqrt{3}+\sqrt{5-2\sqrt{5}}$	$\sqrt{15+6\sqrt{5}}-\sqrt{5}-2$	$\frac{1}{2}\left(\sqrt{50+22\sqrt{5}}-3\sqrt{3}-\sqrt{15}\right)$
$69^\circ = \frac{23}{60}\pi$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(\sqrt{5-2\sqrt{5}}+2-\sqrt{3}\right)$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(2+\sqrt{3}-\sqrt{5-2\sqrt{5}}\right)$	$\frac{1}{4}\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}+1\right)\left(\sqrt{10-2\sqrt{5}}-\sqrt{5}+1\right)$
$72^\circ = \frac{2\pi}{5}$	$\sqrt{5}+1$	$\frac{1}{5}\sqrt{50-10\sqrt{5}}$	$\frac{1}{5}\sqrt{25-10\sqrt{5}}$
$75^\circ = \frac{5\pi}{12}$	$\sqrt{6}+\sqrt{2}$	$\sqrt{6}-\sqrt{2}$	$2-\sqrt{3}$
$78^\circ = \frac{13\pi}{30}$	$\sqrt{5+2\sqrt{5}}+\sqrt{3}$	$\sqrt{15-6\sqrt{5}}-\sqrt{5}+2$	$\frac{1}{2}\left(3\sqrt{3}-\sqrt{15}-\sqrt{50-22\sqrt{5}}\right)$
$81^\circ = \frac{19\pi}{20}$	$\frac{1}{2}\left(3\sqrt{2}+\sqrt{10}+2\sqrt{5}+\sqrt{5}\right)$	$\frac{1}{2}\left(3\sqrt{2}+\sqrt{10}-2\sqrt{5+\sqrt{5}}\right)$	$\sqrt{5}+1-\sqrt{5+2\sqrt{5}}$
$84^\circ = \frac{7\pi}{15}$	$\sqrt{15+6\sqrt{5}}+\sqrt{5}+2$	$\sqrt{3}-\sqrt{5-2\sqrt{5}}$	$\frac{1}{2}\left(\sqrt{10-2\sqrt{5}}-\sqrt{15}+\sqrt{3}\right)$
$87^\circ = \frac{29\pi}{60}$	$\frac{1}{2}\left(\sqrt{10}+\sqrt{6}\right)\left(2+\sqrt{3}+\sqrt{5+2\sqrt{5}}\right)$	$\frac{1}{2}\left(\sqrt{10}-\sqrt{6}\right)\left(\sqrt{5+2\sqrt{5}}-2+\sqrt{3}\right)$	$\frac{1}{4}\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{3}-1\right)\left(\sqrt{10+2\sqrt{5}}-\sqrt{5}-1\right)$
$90^\circ = \frac{\pi}{2}$	—	1	0

Appendix 12

Hyperbolic Trigonometry

Pythagorean Formulæ

$$\cosh^2 x - \sinh^2 x \equiv 1$$
$$\operatorname{sech}^2 x \equiv 1 - \tanh^2 x$$
$$\operatorname{csch}^2 x \equiv \coth^2 x - 1$$

Angle Addition Formulæ

$$\sinh(x \pm y) \equiv \sinh x \cosh y \pm \cosh x \sinh y$$
$$\cosh(x \pm y) \equiv \cosh x \cosh y \pm \sinh x \sinh y$$
$$\tanh(x \pm y) \equiv \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$

Double-Angle Formulæ

$$\sinh 2x \equiv 2 \sinh x \cosh x$$
$$\cosh 2x \equiv \cosh^2 x + \sinh^2 x$$
$$\tanh 2x \equiv \frac{2 \tanh x}{1 + \tanh^2 x}$$

Half-Angle Formulæ

$$\sinh \frac{x}{2} \equiv \pm \sqrt{\frac{\cosh x - 1}{2}}$$
$$\cosh \frac{x}{2} \equiv \sqrt{\frac{\cosh x + 1}{2}}$$
$$\tanh \frac{x}{2} \equiv \frac{\sinh x}{\cosh x + 1} \equiv \frac{\cosh x - 1}{\sinh x}$$

Products of Hyperbolic Sines and Cosines

$$\sinh x \ \cosh y \equiv \frac{1}{2}\left[\sinh(x+y)+\sinh(x-y)\right]$$

$$\cosh x \ \sinh y \equiv \frac{1}{2}\left[\sinh(x+y)-\sinh(x-y)\right]$$

$$\cosh x \ \cosh y \equiv \frac{1}{2}\left[\cosh(x+y)+\cosh(x-y)\right]$$

$$\sinh x \ \sinh y \equiv \frac{1}{2}\left[\cosh(x+y)-\cosh(x-y)\right]$$

Power Reduction Formulæ

$$\sinh^2 x \equiv \tfrac{1}{2}\left(\cosh 2x - 1\right)$$

$$\cosh^2 x \equiv \tfrac{1}{2}\left(\cosh 2x + 1\right)$$

Relations to Plane Trigonometric Functions

$$\sinh x \equiv -i \ \sin(ix)$$

$$\cosh x \equiv \cos(ix)$$

$$\tanh x \equiv -i \ \tan(ix)$$

Appendix 13

Useful Series

The first four series are valid if $|x| < 1$; the fifth is valid for $x^2 < a^2$; and the last three are valid for all real x.

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \frac{7}{256}x^5 - \frac{21}{1024}x^6 + \frac{33}{2048}x^7 - \frac{429}{32768}x^8 + \cdots \quad (13.1)$$

$$(1-x)^{1/2} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \frac{5}{128}x^4 - \frac{7}{256}x^5 - \frac{21}{1024}x^6 - \frac{33}{2048}x^7 - \frac{429}{32768}x^8 - \cdots \quad (13.2)$$

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \frac{35}{128}x^4 - \frac{63}{256}x^5 + \frac{231}{1024}x^6 - \frac{429}{2048}x^7 + \frac{6435}{32768}x^8 - \cdots \quad (13.3)$$

$$(1-x)^{-1/2} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4 + \frac{63}{256}x^5 + \frac{231}{1024}x^6 + \frac{429}{2048}x^7 + \frac{6435}{32768}x^8 + \cdots \quad (13.4)$$

$$\frac{1}{a+x} = \frac{1}{a} - \frac{x}{a^2} + \frac{x^2}{a^3} - \frac{x^3}{a^4} + \frac{x^4}{a^5} - \frac{x^5}{a^6} + \cdots \quad (13.5)$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^8}{40320} + \frac{x^9}{362880} + \cdots \quad (13.6)$$

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880} - \frac{x^{11}}{39916800} + \frac{x^{13}}{6227020800} - \cdots \quad (13.7)$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} - \frac{x^{10}}{3628800} + \frac{x^{12}}{479001600} - \cdots \quad (13.8)$$

Appendix 14

Mathematical Subtleties

- When taking the square root of both sides of an equation, a $\pm$ sign must always be introduced. For example:

  $$x^2 = a \qquad \Rightarrow \qquad x = \pm\sqrt{a}$$

  Both roots may be valid, or, depending on the problem, it may be that one root or the other may be rejected on mathematical or physical grounds.

- Dividing an equation through by a variable may result in losing roots. For example, suppose we have

  $$x^2 - 4x = 0$$

  Dividing through by the variable x will result in one solution, $x = 4$; the solution $x = 0$ has been lost. Instead of dividing through by the variable x, the proper procedure is to *factor out* an x:

  $$x(x - 4) = 0$$

  Since the product on the left-hand side is zero, it follows that either $x = 0$ or $x = 4$, and we retain both roots.

- Likewise, multiplying an equation through by a variable may introduce *new* roots that were not roots of the original equation. For example, given the equation

  $$\frac{3x^2}{x} = 6$$

  Multiplying through by the variable x, we get

  $$3x^2 = 6x$$

  or

  $$x(3x - 6) = 0$$

  so that $x = 0$ or $x = 2$. But clearly $x = 0$ is not a valid solution to the original equation, since it results in $\frac{0}{0}$, which is undefined. The *only* valid root is $x = 2$.

  It is a good practice to try substituting the roots you find back into the original equation to make sure it is satisfied by each of the roots you've found.

- The relation

$$\sqrt{x}\sqrt{y} = \sqrt{xy} \tag{14.1}$$

  is valid only for $x, y \geq 0$.

- Some mathematical conventions:

  ⋆ 1 is *not* considered a prime number.

  ⋆ $0! = 1$

  ⋆ $0^0 = 1$

  ⋆ Towers of exponents are evaluated from the top down: $a^{b^c} = a^{(b^c)}$

- When taking an inverse trigonometric function, there will in general be *two* correct values; your calculator will give only one value, the *principal value* (P.V.). The other value is found using the table below.

Function	P.V.	Other value
arcsin	θ	$\pi - \theta$
arccos	θ	$-\theta$
arctan	θ	$\pi + \theta$
arcsec	θ	$-\theta$
arccsc	θ	$\pi - \theta$
arccot	θ	$\pi + \theta$

For arctan (y/x), add π to the calculator's principal value answer if $x < 0$.

Appendix 15

Table of Derivatives

$$\frac{d}{dx}\,a = 0 \tag{15.1}$$

$$\frac{d}{dx}\,x = 1 \tag{15.2}$$

$$\frac{d}{dx}\,x^n = nx^{n-1} \tag{15.3}$$

$$\frac{d}{dx}\,\sqrt{x} = \frac{1}{2\sqrt{x}} \tag{15.4}$$

$$\frac{d}{dx}\,\sin x = \cos x \tag{15.5}$$

$$\frac{d}{dx}\,\cos x = -\sin x \tag{15.6}$$

$$\frac{d}{dx}\,\tan x = \sec^2 x \tag{15.7}$$

$$\frac{d}{dx}\,\sec x = \tan x \sec x \tag{15.8}$$

$$\frac{d}{dx}\,\csc x = -\cot x \csc x \tag{15.9}$$

$$\frac{d}{dx}\,\cot x = -\csc^2 x \tag{15.10}$$

$$\tag{15.11}$$

$$\frac{d}{dx} e^x = e^x \tag{15.12}$$

$$\frac{d}{dx} \ln x = \frac{1}{x} \tag{15.13}$$

$$\frac{d}{dx} a^x = a^x \ln a \tag{15.14}$$

$$\frac{d}{dx} \log_a x = \frac{1}{x \ln a} \tag{15.15}$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}} \tag{15.16}$$

$$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1 - x^2}} \tag{15.17}$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2} \tag{15.18}$$

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2 - 1}} \tag{15.19}$$

$$\frac{d}{dx} \csc^{-1} x = \frac{-1}{|x|\sqrt{x^2 - 1}} \tag{15.20}$$

$$\frac{d}{dx} \cot^{-1} x = \frac{-1}{1 + x^2} \tag{15.21}$$

$$\frac{d}{dx} \sinh x = \cosh x \tag{15.22}$$

$$\frac{d}{dx} \cosh x = \sinh x \tag{15.23}$$

$$\frac{d}{dx} \tanh x = \operatorname{sech}^2 x \tag{15.24}$$

Appendix 16

Table of Integrals

In the following table, an arbitrary constant C should be added to each result.

$$\int dx = x \tag{16.1}$$

$$\int a\, dx = ax \tag{16.2}$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1) \tag{16.3}$$

$$\int \sqrt{x}\, dx = \frac{2}{3}\sqrt{x^3} \tag{16.4}$$

$$\int \frac{1}{x}\, dx = \ln|x| \tag{16.5}$$

$$\int \sin x\, dx = -\cos x \tag{16.6}$$

$$\int \cos x\, dx = \sin x \tag{16.7}$$

$$\int \tan x\, dx = \ln|\sec x| \tag{16.8}$$

(16.9)

$$\int \sec x \; dx = \ln|\sec x + \tan x| \tag{16.10}$$

$$\int \csc x \; dx = \ln|\csc x - \cot x| \tag{16.11}$$

$$\int \cot x \; dx = \ln|\sin x| \tag{16.12}$$

$$\int e^x \; dx = e^x \tag{16.13}$$

$$\int \ln x \; dx = x \ln x - x \tag{16.14}$$

$$\int a^x \; dx = \frac{a^x}{\ln a} \tag{16.15}$$

$$\int \log_a x \; dx = \frac{x \ln x - x}{\ln a} \tag{16.16}$$

$$\int \sinh x \; dx = \cosh x \tag{16.17}$$

$$\int \cosh x \; dx = \sinh x \tag{16.18}$$

$$\int \tanh x \; dx = \ln \cosh x \tag{16.19}$$

Appendix 17

Fundamental Theorems of Mathematics

Fundamental Theorem of Arithmetic

Every integer greater than 1 can by represented uniquely as a product of prime numbers.

Fundamental Theorem of Algebra

Every polynomial equation of degree n has n (complex) roots.

Fundamental Theorem of Calculus

Differentiation and integration are inverse operations of each other.

Appendix 18

Chi-Squared Table

The following χ^2 table shows critical values for the χ^2 test for various degrees of freedom (rows). The columns show the probability of exceeding the confidence level and therefore rejecting the null hypothesis. Nominally we will use a value of 0.05 for this (column highlighted in green) so that we find the critical value for accepting the null hypothesis with 95% confidence.

Critical values of chi-square (right tail)

Significance level (α)

Degrees of freedom (*df*)	.99	.975	.95	.9	.1	.05	.025	.01
1	-------	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932
22	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642
27	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278
29	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588
30	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892
40	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691
50	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154
60	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379
70	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425
80	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329
100	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116
1000	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807

Appendix 19

Index to "Mathematical Games"

The following is an index to the 298 "Mathematical Games" columns by Martin Gardner, appearing in *Scientific American* magazine between 1956 and 1986.

Issue		Book	Topic
Dec	1956	1	Hexaflexagons
Jan	1957	1	Magic with a Matrix
Feb	1957	1	Nine Problems
Mar	1957	1	Ticktacktoe
Apr	1957	1	Probability Paradoxes
May	1957	1	The Icosian Game and the Tower of Hanoi
Jun	1957	1	Curious Topological Models
Jul	1957	1	The Game of Hex
Aug	1957	1	Sam Loyd: America's Greatest Puzzlist
Sep	1957	1	Mathematical Card Tricks
Oct	1957	1	Memorizing Numbers
Nov	1957	1	Nine More Problems
Dec	1957	1	Polyominoes
Jan	1958	1	Fallacies
Feb	1958	1	Nim and Tac Tix
Mar	1958	1	Left or Right?
Apr	1958	2	The Monkey and the Coconuts
May	1958	2	Tetraflexagons
Jun	1958	2	Henry Ernest Dudeney: England's Greatest Puzzlist
Jul	1958	2	Digital Roots
Aug	1958	2	Nine Problems
Sep	1958	2	The Soma Cube
Oct	1958	2	Recreational Topology
Nov	1958	2	Squaring the Square
Dec	1958	2	The Five Platonic Solids

Issue		Book	Topic
Jan	1959	2	Mazes
Feb	1959	2	Recreational Logic
Mar	1959	2	Magic Squares
Apr	1959	2	James Hugh Riley Shows, Inc.
May	1959	2	Nine More Problems
Jun	1959	2	Eleusis: The Induction Game
Jul	1959	2	Origami
Aug	1959	2	Phi: The Golden Ratio
Sep	1959	2	Mechanical Puzzles
Oct	1959	2	Probability and Ambiguity
Nov	1959	3	Euler's Spoilers: The Discovery of an Order-10 Graeco-Latin Square
Dec	1959	3	Group Theory and Braids
Jan	1960	2	The Mysterious Dr. Matrix
Feb	1960	3	Eight Problems
Mar	1960	3	The Games and Puzzles of Lewis Carroll
Apr	1960	3	Board Games
May	1960	3	Packing Spheres
Jun	1960	3	Paper Cutting
Jul	1960	3	The Transcendental Number Pi
Aug	1960	3	Victor Eigen: Mathemagician
Sep	1960	3	The Four-Color Map Theorem
Oct	1960	3	Nine Problems
Nov	1960	3	Polyominoes and Fault-Free Rectangles
Dec	1960	3	The Binary System
Jan	1961	9	Dr. Matrix (Los Angeles)
Feb	1961	3	The Ellipse
Mar	1961	3	The 24 Color Squares and the 30 Color Cubes
Apr	1961	3	H.S.M. Coxeter
May	1961	3	Mr. Apollinax Visits New York
Jun	1961	3	Nine More Problems
Jul	1961	3	Bridg-it and Other Games
Aug	1961	3	The Calculus of Finite Differences
Sep	1961	4	Knots and Borromean Rings
Oct	1961	4	The Transcendental Number *e*
Nov	1961	4	Geometric Dissections
Dec	1961	4	Scarne on Gambling

Issue		Book	Topic
Jan	1962	4	The Church of the Fourth Dimension
Feb	1962	4	Eight Problems
Mar	1962	4	A Matchbox Game-Learning Machine
Apr	1962	4	Spirals
May	1962	4	Rotations and Reflections
Jun	1962	4	Peg Solitaire
Jul	1962	4	Flatlands
Aug	1962	4	Chicago Magic Convention
Sep	1962	4	Tests of Divisibility
Oct	1962	4	Nine Problems
Nov	1962	4	The Eight Queens and Other Chessboard Diversions
Dec	1962	4	A Loop of String
Jan	1963	9	Dr. Matrix (Sing Sing)
Feb	1963	4	Curves of Constant Width
Mar	1963	4	The Paradox of the Unexpected Hanging
Apr	1963	4	Thirty-Seven Catch Questions
May	1963	4	Rep-Tiles: Replicating Figures on the Plane
Jun	1963	5	The Helix
Jul	1963	5	Klein Bottles and Other Surfaces
Aug	1963	5	Combinatorial Theory
Sep	1963	5	Bouncing Balls in Polygons and Polyhedrons
Oct	1963	5	Four Unusual Board Games
Nov	1963	5	The Rigid Square and Eight Other Problems
Dec	1963	5	Parity Checks
Jan	1964	9	Dr. Matrix (Chicago)
Feb	1964	5	Sliding-Block Puzzles
Mar	1964	5	Patterns and Primes
Apr	1964	5	Graph Theory
May	1964	5	The Ternary System
Jun	1964	5	The Trip around the Moon and Seven Other Problems
Jul	1964	5	The Cycloid: Helen of Geometry
Aug	1964	5	Mathematical Magic Tricks
Sep	1964	5	Word Play
Oct	1964	5	The Pythagorean Theorem
Nov	1964	5	Limits of Infinite Series
Dec	1964	5	Polyiamonds

Issue		Book	Topic
Jan	1965	9	Dr. Matrix (Miami Beach)
Feb	1965	5	Tetrahedrons
Mar	1965	5	Coleridge's Apples and Eight Other Problems
Apr	1965	5	Infinite Regress
May	1965	5	The Lattice of Integers
Jun	1965	5	O'Gara, the Mathematical Mailman
Jul	1965	5	Op Art
Aug	1965	5	Extraterrestrial Communication
Sep	1965	6	Piet Hein's Superellipse
Oct	1965	7	Polyominoes and Rectification
Nov	1965	6	The Red-Faced Cube and Other Problems
Dec	1965	6	Magic Stars and Polyhedrons
Jan	1966	9	Dr. Matrix (Philadelphia)
Feb	1966	6	Penny Puzzles
Mar	1966	6	Aleph-null and Aleph-one
Apr	1966	6	The Art of M. C. Escher
May	1966	6	Cooks and Quibble-Cooks
Jun	1966	6	How to Trisect an Angle
Jul	1966	6	The Numerology of Dr. Fliess
Aug	1966	6	The Rising Hourglass and Other Physics Puzzles
Sep	1966	6	Mrs. Perkins' Quilt and Other Square-Packing Problems
Oct	1966	6	Card Shuffles
Nov	1966	6	Hypercubes
Dec	1966	6	Pascal's Triangle
Jan	1967	9	Dr. Matrix (Wordsmith College)
Feb	1967	6	Jam, Hot, and Other Games
Mar	1967	7	The Dragon Curve and Other Problems
Apr	1967	6	Calculating Prodigies
May	1967	6	Tricks of Lightning Calculators
Jun	1967	7	Polyhexes and Polyaboloes
Jul	1967	6	Sprouts and Brussels Sprouts
Aug	1967	7	Factorial Oddities
Sep	1967	7	Double Acrostics
Oct	1967	7	Knights of the Square Table
Nov	1967	7	The Cocktail Cherry and Other Problems
Dec	1967	7	Game Theory, Guess It, Foxholes

Issue		Book	Topic
Jan	1968	9	Dr. Matrix (Squaresville)
Feb	1968	7	Trees
Mar	1968	7	Perfect, Amicable, Sociable
Apr	1968	8	Dollar Bills
May	1968	8	Spheres and Hyperspheres
Jun	1968	7	Playing Cards
Jul	1968	6	Random Numbers
Aug	1968	7	Ridiculous Questions
Sep	1968	7	Finger Arithmetic
Oct	1968	7	Colored Triangles and Cubes
Nov	1968	7	Dice
Dec	1968	7	Möbius Bands
Jan	1969	9	Dr. Matrix (Fifth Avenue)
Feb	1969	8	Boolean Algebra
Mar	1969	8	Fibonacci and Lucas Numbers
Apr	1969	8	The Rotating Round Table and Other Problems
May	1969	8	Random Walks and Gambling
Jun	1969	8	Random Walks on the Plane and in Space
Jul	1969	8	Matches
Aug	1969	8	Simplicity
Sep	1969	8	Mascheroni Constructions
Oct	1969	9	Dr. Matrix (The Moon)
Nov	1969	8	Patterns of Induction
Dec	1969	8	Dominoes
Jan	1970	8	The Abacus
Feb	1970	8	Eccentric Chess and Other Problems
Mar	1970	8	Cyclic Numbers
Apr	1970	8	Solar System Oddities
May	1970	8	Optical Illusions
Jun	1970	8	Elegant Triangles
Jul	1970	10	Diophantine Analysis and Fermat's Last Theorem
Aug	1970	8	Palindromes: Words and Numbers
Sep	1970	10	Wheels
Oct	1970	10	The Game of Life, Part I
Nov	1970	10	The Knotted Molecule and Other Problems
Dec	1970	10	Nontransitive Dice and Other Probability Paradoxes

Issue		Book	Topic
Jan	1971	9	Dr. Matrix (Honolulu)
Feb	1971	10	The Game of Life, Part II
Mar	1971	10	Alephs and Supertasks
Apr	1971	10	Geometric Fallacies
May	1971	10	The Combinatorics of Paper Folding
Jun	1971	8	Can Machines Think?
Jul	1971	10	A Set of Quickies
Aug	1971	10	Ticktacktoe Games
Sep	1971	10	Plaiting Polyhedrons
Oct	1971	10	The Game of Halma
Nov	1971	10	Advertising Premiums
Dec	1971	10	Salmon on Austin's Dog
Jan	1972	10	Nim and Hackenbush
Feb	1972	9	Dr. Matrix (Houston)
Mar	1972	10	Golomb's Graceful Graphs
Apr	1972	10	Charles Addams' Skier and Other Problems
May	1972	10	Chess Tasks
Jun	1972	10	Slither, $3X + 1$, and Other Curious Questions
Jul	1972	10	Mathematical Tricks with Cards
Aug	1972	11	The Binary Gray Code
Sep	1972	11	Polycubes
Oct	1972	11	Coincidence
Nov	1972	11	Bacon's Cipher
Dec	1972	11	Doughnuts: Linked and Knotted
Jan	1973	11	Sim, Chomp, and Race Track
Feb	1973	11	Elevators
Mar	1973	11	Napier's Bones
Apr	1973	11	Napier's Abacus
May	1973	11	The Tour of the Arrows and Other Problems
Jun	1973	11	Crossing Numbers
Jul	1973	11	Newcomb's Paradox
Aug	1973	9	Dr. Matrix (Clairvoyance Test)
Sep	1973	11	Point Sets on the Sphere
Oct	1973	11	Look-See Proofs
Nov	1973	11	Worm Paths
Dec	1973	11	Waring's Problems

Issue		Book	Topic
Jan	1974	11	The I Ching
Feb	1974	11	Cram, Bynum and Quadraphage
Mar	1974	11	Reflections on Newcomb's Paradox
Apr	1974	11	Reverse the Fish and Other Problems
May	1974	12	Time Travel
Jun	1974	9	Dr. Matrix (Pyramid Lake)
Jul	1974	12	Hexes and Stars
Aug	1974	12	Tangrams, Part 1
Sep	1974	12	Tangrams, Part 2
Oct	1974	12	Nontransitive Paradoxes
Nov	1974	12	Combinatorial Card Problems
Dec	1974	12	Melody-Making Machines
Jan	1975	12	Anamorphic Art
Feb	1975	7	Nothing
Mar	1975	12	The Rubber Rope and Other Problems
Apr	1975	12	Six Sensational Discoveries (April Fools)
May	1975	12	The Császár Polyhedron
Jun	1975	12	Dodgem and Other Simple Games
Jul	1975	12	Tiling with Convex Polygons
Aug	1975	12	Tiling with Polyominoes, Polyiamonds, and Polyhexes
Sep	1975	9	Dr. Matrix (The King James Bible)
Oct	1975	*	Concerning an Effort to Demonstrate ESP by Machine
Nov	1975	12	Curious Maps
Dec	1975	12	The Sixth Symbol and Other Problems
Jan	1976	12	Magic Squares and Cubes
Feb	1976	12	Block Packing
Mar	1976	12	Induction and Probability
Apr	1976	15	Trivalent Graphs, Snarks, and Boojums
May	1976	7	Everything
Jun	1976	12	Catalan Numbers
Jul	1976	12	Fun with a Pocket Calculator
Aug	1976	12	Tree-Plant Problems
Sep	1976	13	Conway's Surreal Numbers
Oct	1976	13	Back from the Klondike and Other Problems
Nov	1976	9	Dr. Matrix (Calcutta)
Dec	1976	13	Mandelbrot's Fractals

Issue		Book	Topic
Jan	1977	13	Penrose Tiling
Feb	1977	13	The Oulipo
Mar	1977	13	Wythoff's Nim
Apr	1977	13	Pool-Ball Triangles and Other Problems
May	1977	13	Mathematical Induction and Colored Hats
Jun	1977	13	Negative Numbers
Jul	1977	13	Cutting Shapes into N Congruent Parts
Aug	1977	13	Trapdoor Ciphers
Sep	1977	13	Hyperbolas
Oct	1977	13	The New Eleusis
Nov	1977	13	Ramsey Theory
Dec	1977	9	Dr. Matrix (Stanford)
Jan	1978	13	From Burrs to Berrocal
Feb	1978	13	Sicherman Dice, the Kruskal Count and Other Curiosities
Mar	1978	13	Raymond Smyllyan's Logic Puzzles
Apr	1978	14	White, Brown, and Fractal Music
May	1978	14	The Tinkly Temple Bells
Jun	1978	14	Mathematical Zoo
Jul	1978	14	Charles Sanders Peirce
Aug	1978	14	Twisted Prismatic Rings
Sep	1978	14	The Thirty Color Cubes
Oct	1978	14	Egyptian Fractions
Nov	1978	14	Minimal Sculpture
Dec	1978	9	Dr. Matrix (Chautauqua)
Jan	1979	14	Tangent Circles
Feb	1979	14	The Rotating Table and Other Problems
Mar	1979	14	Does Time Ever Stop? Can the Past Be Altered?
Apr	1979	14	Generalized Ticktacktoe
May	1979	14	Psychic Wonders and Probability
Jun	1979	14	Mathematical Chess Problems
Jul	1979	14	Douglas Hofstadter's *Gödel, Escher, Bach*
Aug	1979	14	Imaginary Numbers
Sep	1979	14	Pi and Poetry: Some Accidental Patterns
Oct	1979	14	Packing Squares
Nov	1979	14	Chaitin's Omega
Dec	1979	15	A Toroidal Paradox and Other Problems

Issue		Book	Topic
Jan	1980	15	Checker Recreations
Feb	1980	15	M-Pire Maps
Mar	1980	15	Directed Graphs and Cannibals
Apr	1980	15	Fun with Eggs
May	1980	15	Dinner Guests, Schoolgirls, and Handcuffed Prisoners
Jun	1980	15	The Monster and Other Sporadic Groups
Jul	1980	15	The Wonders of a Planiverse
Aug	1980	15	The Power of the Pigeonhole
Sep	1980	9	Dr. Matrix (Istanbul)
Oct	1980	15	Voting Mathematics
Nov	1980	15	Taxicab Geometry
Dec	1980	15	Strong Laws of Small Primes
Feb	1981	15	Modulo Arithmetic and Hummer's Wicked Witch
Apr	1981	15	Levina Seeks a Room and Other Problems
Jun	1981	15	The Symmetry Creations of Scott Kim
Aug	1981	15	Parabolas
Oct	1981	15	Non-Euclidean Geometry
Dec	1981	11	The Laffer Curve
Aug	1983	15	Bulgarian Solitaire and Other Seemingly Endless Tasks
Sep	1983	15	The Topology of Knots
Jun	1986	15	Minimal Steiner Trees

**Note:* The October 1975 column was not published in any of the books.

Book titles

1. Hexaflexagons and Other Mathematical Diversions: The First *Scientific American* Book of Mathematical Puzzles and Games
2. The Second *Scientific American* Book of Mathematical Puzzles and Diversions
3. New Mathematical Diversions
4. The Unexpected Hanging and Other Mathematical Diversions: A Classic Collection of Puzzles and Games from *Scientific American*
5. Martin Gardner's 6th Book of Mathematical Diversions from *Scientific American*
6. Mathematical Carnival
7. Mathematical Magic Show
8. Mathematical Circus
9. The Magic Numbers of Dr. Matrix

10. Wheels, Life, and Other Mathematical Amusements
11. Knotted Doughnuts and Other Mathematical Entertainments
12. Time Travel and Other Mathematical Bewilderments
13. Penrose Tiles to Trapdoor Ciphers...and the Return of Dr. Matrix
14. Fractal Music, Hypercards, and More: Mathematical Recreations from *Scientific American* Magazine
15. The Last Recreations: Hydras, Eggs, and Other Mathematical Mystifications

Appendix 20

SI Units

Table 20-1. SI base units.

Name	Symbol	Quantity
meter	m	length
kilogram	kg	mass
second	s	time
ampere	A	electric current
kelvin	K	temperature
mole	mol	amount of substance
candela	cd	luminous intensity

Table 20-2. Derived SI units.

Name	Symbol	Definition	Base Units	Quantity
radian	rad	m / m	—	plane angle
steradian	sr	m^2 / m^2	—	solid angle
newton	N	kg m s^{-2}	kg m s^{-2}	force
joule	J	N m	kg m^2 s^{-2}	energy
watt	W	J / s	kg m^2 s^{-3}	power
pascal	Pa	N / m^2	kg m^{-1} s^{-2}	pressure
hertz	Hz	s^{-1}	s^{-1}	frequency
coulomb	C	A s	A s	electric charge
volt	V	J / C	kg m^2 A^{-1} s^{-3}	electric potential
ohm	Ω	V / A	kg m^2 A^{-2} s^{-3}	electrical resistance
siemens	S	A / V	kg^{-1} m^{-2} A^2 s^3	electrical conductance
farad	F	C / V	kg^{-1} m^{-2} A^2 s^4	capacitance
weber	Wb	V s	kg m^2 A^{-1} s^{-2}	magnetic flux
tesla	T	Wb / m^2	kg A^{-1} s^{-2}	magnetic induction
henry	H	Wb / A	kg m^2 A^{-2} s^{-2}	induction
lumen	lm	cd sr	cd sr	luminous flux
lux	lx	lm / m^2	cd sr m^{-2}	illuminance
becquerel	Bq	s^{-1}	s^{-1}	radioactivity
gray	Gy	J / kg	m^2 s^{-2}	absorbed dose
sievert	Sv	J / kg	m^2 s^{-2}	dose equivalent
katal	kat	mol / s	mol s^{-1}	catalytic activity

Table 20-3. SI prefixes.

Prefix	Symbol	Definition	English
quetta-	Q	10^{30}	nonillion
ronna-	R	10^{27}	octillion
yotta-	Y	10^{24}	septillion
zetta-	Z	10^{21}	sextillion
exa-	E	10^{18}	quintillion
peta-	P	10^{15}	quadrillion
tera-	T	10^{12}	trillion
giga-	G	10^{9}	billion
mega-	M	10^{6}	million
kilo-	k	10^{3}	thousand
hecto-	h	10^{2}	hundred
deka-	da	10^{1}	ten
deci-	d	10^{-1}	tenth
centi-	c	10^{-2}	hundredth
milli-	m	10^{-3}	thousandth
micro-	μ	10^{-6}	millionth
nano-	n	10^{-9}	billionth
pico-	p	10^{-12}	trillionth
femto-	f	10^{-15}	quadrillionth
atto-	a	10^{-18}	quintillionth
zepto-	z	10^{-21}	sextillionth
yocto-	y	10^{-24}	septillionth
ronto-	r	10^{-27}	octillionth
quecto-	q	10^{-30}	nonillionth

Table 20-4. Prefixes for *computer use only*.

Prefix	Symbol	Definition	
yobi-	Yi	2^{80}	= 1,208,925,819,614,629,174,706,176
zebi-	Zi	2^{70}	= 1,180,591,620,717,411,303,424
exbi-	Ei	2^{60}	= 1,152,921,504,606,846,976
pebi-	Pi	2^{50}	= 1,125,899,906,842,624
tebi-	Ti	2^{40}	= 1,099,511,627,776
gibi-	Gi	2^{30}	= 1,073,741,824
mebi-	Mi	2^{20}	= 1,048,576
kibi-	Ki	2^{10}	= 1,024

Appendix 21

Gaussian Units

Table 21-1. Gaussian base units.

Name	Symbol	Quantity
centimeter	cm	length
gram	g	mass
second	s	time
kelvin	K	temperature
mole	mol	amount of substance
candela	cd	luminous intensity

Table 21-2. Derived Gaussian units.

Name	Symbol	Definition	Base Units	Quantity
radian	rad	m / m	—	plane angle
steradian	sr	m^2 / m^2	—	solid angle
dyne	dyn	g cm s^{-2}	g cm s^{-2}	force
erg	erg	dyn cm	g cm^2 s^{-2}	energy
statwatt	statW	erg / s	g cm^2 s^{-3}	power
barye	ba	dyn / cm^2	g cm^{-1} s^{-2}	pressure
galileo	Gal	cm / s^2	cm s^{-2}	acceleration
poise	P	g / (cm s)	g cm^{-1} s^{-1}	dynamic viscosity
stokes	St	cm^2 / s	cm^2 s^{-1}	kinematic viscosity
hertz	Hz	s^{-1}	s^{-1}	frequency
statcoulomb	statC		$g^{1/2}$ $cm^{3/2}$ s^{-1}	electric charge
franklin	Fr	statC	$g^{1/2}$ $cm^{3/2}$ s^{-1}	electric charge
statampere	statA	statC / s	$g^{1/2}$ $cm^{3/2}$ s^{-2}	electric current
statvolt	statV	erg / statC	$g^{1/2}$ $cm^{1/2}$ s^{-1}	electric potential
statohm	statΩ	statV / statA	s cm^{-1}	electrical resistance
statfarad	statF	statC / statV	cm	capacitance
maxwell	Mx	statV cm	$g^{1/2}$ $cm^{3/2}$ s^{-1}	magnetic flux
gauss	G	Mx / cm^2	$g^{1/2}$ $cm^{-1/2}$ s^{-1}	magnetic induction
oersted	Oe	statA s / cm^2	$g^{1/2}$ $cm^{-1/2}$ s^{-1}	magnetic intensity
gilbert	Gb	statA	$g^{1/2}$ $cm^{3/2}$ s^{-2}	magnetomotive force
unit pole	pole	dyn / Oe	$g^{1/2}$ $cm^{3/2}$ s^{-1}	magnetic pole strength
stathenry	statH	erg / $statA^2$	s^2 cm^{-1}	induction
lumen	lm	cd sr	cd sr	luminous flux
phot	ph	lm / cm^2	cd sr cm^{-2}	illuminance
stilb	sb	cd / cm^2	cd cm^{-2}	luminance
lambert	Lb	$1/\pi$ cd / cm^2	cd cm^{-2}	luminance
kayser	K	1 / cm	cm^{-1}	wave number
becquerel	Bq	s^{-1}	s^{-1}	radioactivity
katal	kat	mol / s	mol s^{-1}	catalytic activity

Appendix 22

British Engineering Units

Table 22-1. Britsh Engineering base units.

Name	Symbol	Quantity
foot	ft	length
slug	slug	mass
second	s	time
degree Rankine	°R	temperature
pound-mole	lb-mol	amount of substance
candle	candle	luminous intensity

Table 22-2. Derived British Engineering units.

Name	Symbol	Definition	Base Units	Quantity
radian	rad	ft / ft	—	plane angle
steradian	sr	ft^2 / ft^2	—	solid angle
pound-force	lbf	slug ft s^{-2}	slug ft s^{-2}	force
hertz	Hz	s^{-1}	s^{-1}	frequency
becquerel	Bq	s^{-1}	s^{-1}	radioactivity

Appendix 23

Physical Constants

Table 23-1. Fundamental physical constants (CODATA 2018).

Description	Symbol	Value
Speed of light (vacuum)	c	2.99792458×10^{8} m/s
Gravitational constant	G	6.67430×10^{-11} m^3 kg^{-1} s^{-2}
Elementary charge	e	$1.602176634 \times 10^{-19}$ C
Permittivity of free space	ε_0	$8.8541878128 \times 10^{-12}$ F/m
Permeability of free space	μ_0	1.2566370621210^{-6} N/A^2
Coulomb constant $(1/(4\pi\varepsilon_0))$	k_c	$8.9875517923 \times 10^{9}$ m/F
Electron mass	m_e	$9.1093837015 \times 10^{-31}$ kg
Proton mass	m_p	$1.67262192369 \times 10^{-27}$ kg
Neutron mass	m_n	$1.67492749804 \times 10^{-27}$ kg
Atomic mass unit (amu)	u	$1.66053906660 \times 10^{-27}$ kg
Planck constant	h	$6.62607015 \times 10^{-34}$ J s
Planck constant $\div 2\pi$	$\hbar$	$1.0545718176461564 \times 10^{-34}$ J s
Boltzmann constant	k_B	1.380649×10^{-23} J/K
Avogadro constant	N_A	$6.02214076 \times 10^{23}$ mol^{-1}

Table 23-2. Other physical constants.

Description	Symbol	Value
Acceleration due to gravity at Earth surface	g	9.80 m/s^2
Radius of the Earth (eq.)	$R_\oplus$	6378.140 km
Mass of the Earth	$M_\oplus$	5.97320×10^{24} kg
Earth gravity constant	$GM_\oplus$	3.986005×10^{14} m^3 s^{-2}
Speed of sound in air (20°C)	v_{snd}	343 m/s
Density of air (sea level)	ρ_{air}	1.29 kg/m^3
Density of water	ρ_{w}	1 g/cm^3 = 1000 kg/m^3
Index of refraction of water	n_{w}	1.33
Resistivity of copper (20°C)	ρ_{Cu}	1.68×10^{-8} Ω m

Appendix 24

Astronomical Data

Table 24-1. Astronomical constants.

Description	Symbol	Value
Astronomical unit	AU	$1.49597870 \times 10^{11}$ m
Obliquity of ecliptic (J2000)	ε	$23\overset{\circ}{.}4392911$
Solar mass	$M_\odot$	1.9891×10^{30} kg
Solar radius	$R_\odot$	696,000 km
Earth grav. const.	$GM_\oplus$	$3.986004415 \times 10^{14}$ m^3 s^{-2}
Sun grav. const.	$GM_\odot$	$1.32712440041 \times 10^{20}$ m^3 s^{-2}

Table 24-2. Planetary Data.

Planet	Mass (Yg)	Eq. radius (km)	Orbit semi-major axis (Gm)
Mercury	330.2	2439.7	57.91
Venus	4868.5	6051.8	108.21
Earth	5973.6	6378.1	149.60
Mars	641.85	3396.2	227.92
Jupiter	1,898,600	71,492	778.57
Saturn	568,460	60,268	1433.53
Uranus	86,832	25,559	2872.46
Neptune	102,430	24,764	4495.06
Pluto	12.5	1195	5906.38

Appendix 25

Unit Conversion Tables

Time

1 day = 24 hours = 1440 minutes = 86400 seconds
1 hour = 60 minutes = 3600 seconds
1 year = 31 557 600 seconds $\approx \pi \times 10^7$ seconds

Length

1 mile = 8 furlongs = 80 chains = 320 rods = 1760 yards = 5280 feet = 1.609344 km
1 yard = 3 feet = 36 inches = 0.9144 meter
1 foot = 12 inches = 0.3048 meter
1 inch = 2.54 cm
1 nautical mile = 1852 meters = 1.15077944802354 miles
1 fathom = 6 feet
1 parsec = 3.26156376188 light-years = 206264.806245 AU = $3.08567756703 \times 10^{16}$ meters
1 ångström = 0.1 nm = 10^5 fermi = 10^{-10} meter

Mass

1 kilogram = 2.20462262184878 lb
1 pound = 16 oz = 0.45359237 kg
1 slug = 32.1740485564304 lb = 14.5939029372064 kg
1 short ton = 2000 lb
1 long ton = 2240 lb
1 metric ton = 1000 kg

Velocity

15 mph = 22 fps
1 mph = 0.44704 m/s
1 knot = 1.15077944802354 mph = 0.514444444444444 m/s

Area

1 acre = 43560 ft^2 = 4840 yd^2 = 4046.8564224 m^2
1 mile^2 = 640 acres = 2.589988110336 km^2
1 are = 100 m^2
1 hectare = 10^4 m^2 = 2.47105381467165 acres

Volume

1 liter = 1 dm^3 = 10^{-3} m^3 $\approx$ 1 quart
1 m^3 = 1000 liters
1 cm^3 = 1 mL
1 ft^3 = 1728 in^3 = 7.48051948051948 gal = 28.316846592 liters
1 gallon = 231 in^3 = 4 quarts = 8 pints = 16 cups = 3.785411784 liters
1 cup = 8 floz = 16 tablespoons = 48 teaspoons
1 tablespoon = 3 teaspoons = 4 fluidrams
1 dry gallon = 268.8025 in^3 = 4.40488377086 liters
1 imperial gallon = 4.54609 liters
1 bushel = 4 pecks = 8 dry gallons

Density

1 g/cm^3 = 1000 kg/m^3 = 8.34540445201933 lb/gal = 1.043175556502416 lb/pint

Force

1 lbf = 4.44822161526050 newtons = 32.1740485564304 poundals
1 newton = 10^5 dynes

Energy

1 calorie = 4.1868 joules
1 BTU = 1055.05585262 joules
1 ft-lb = 1.35581794833140 joules
1 kW-hr = 3.6 MJ
1 eV = $1.602176634 \times 10^{-19}$ joules
1 joule = 10^7 ergs

Power

1 horsepower = 745.69987158227022 watts
1 statwatt = 1 abwatt = 1 erg/s = 10^{-7} watt

Angle

$\text{rad} = \text{deg} \times \frac{\pi}{180}$ $\text{deg} = \text{rad} \times \frac{180}{\pi}$
1 deg = 60 arcmin = 3600 arcsec

Temperature

$°C = (°F - 32) \times \frac{5}{9}$ $\qquad$ $°F = \left(°C \times \frac{9}{5}\right) + 32$
$K = °C + 273.15$
$°R = °F + 459.67$

Pressure

1 atm = 101325 Pa = 1.01325 bar = 1013.25 millibar = 760 torr
= 760 mmHg = 29.9212598425197 inHg = 14.6959487755134 psi
= 2116.21662367394 lb/ft^2 = 1.05810831183697 ton/ft^2
= 1013250 dyne/cm^2 = 1013250 barye

Electromagnetism

1 statcoulomb = $3.335640951981520 \times 10^{-10}$ coulomb
1 abcoulomb = 10 coulombs
1 statvolt = 299.792458 volts
1 abvolt = 10^{-8} volt
1 maxwell = 10^{-8} weber
1 gauss = 10^{-4} tesla
1 oersted = $250/\pi$ (= 79.5774715459477) A/m

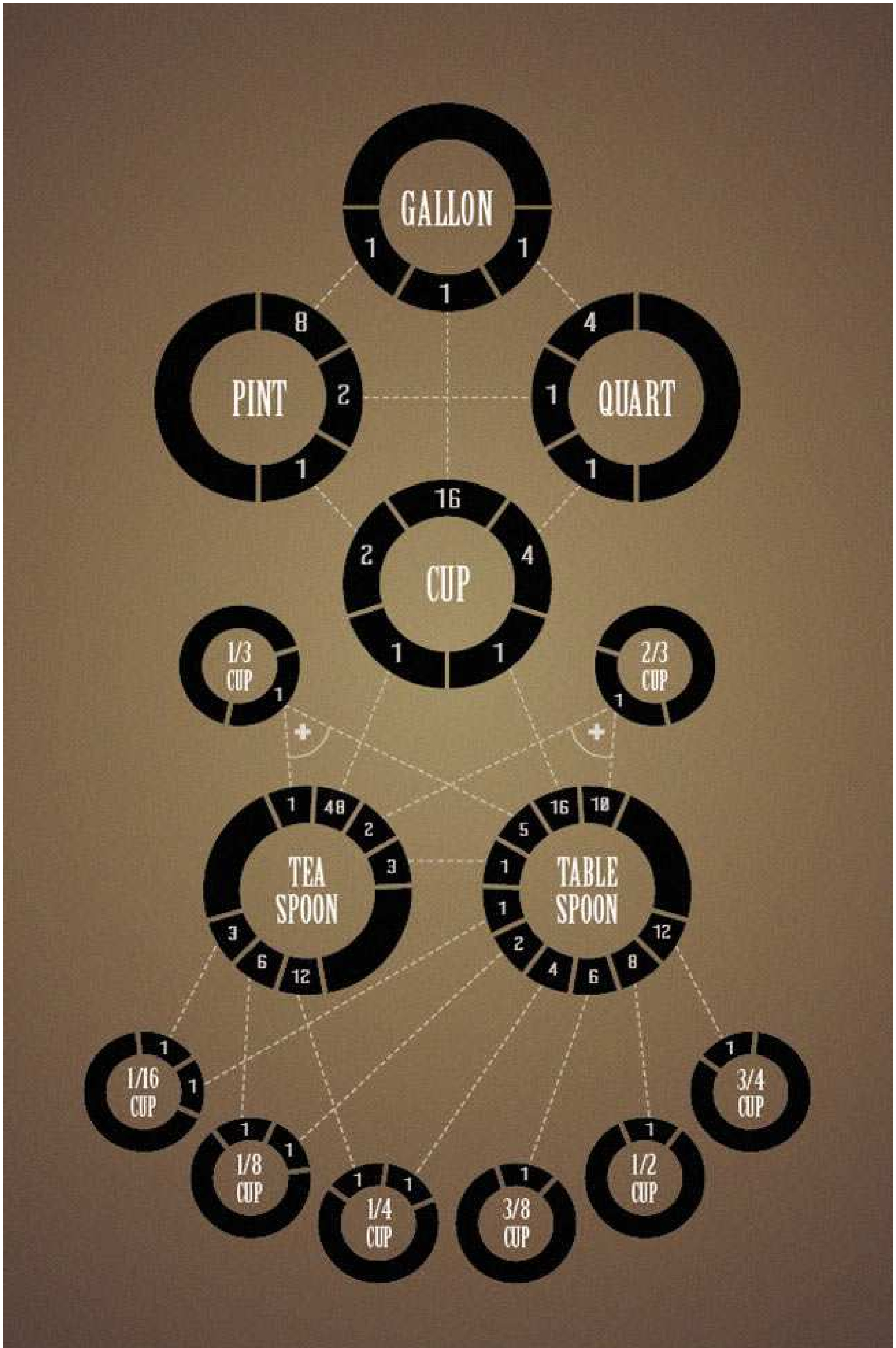


Figure 25.1: Conversion chart for kitchen measurements. (*Credit: S.B. Lattin Design.*)

Appendix 26

Angular Measure

Plane Angle

The most common unit of measure for plane angle is the *degree* ($^\circ$), which is 1/360 of a full circle. Therefore a circle is 360°, a semicircle is 180°, and a right angle is 90°.

A similar unit (seldom used nowadays) is a sort of "metric" angle called the *grad*, defined so that a right angle is 100 grads, and so a full circle is 400 grads.

The SI unit of plane angle is the *radian* (rad), which is defined to be the angle that subtends an arc length equal to the radius of the circle. By this definition, a full circle subtends an angle equal to the arc length of a full circle ($2\pi r$) divided by its radius r — and so a full circle is 2π radians.

Since a hemisphere is 180° or π radians, the conversion factors are:

$$\text{rad} = \frac{\pi}{180} \times \text{deg} \tag{26.1}$$

$$\text{deg} = \frac{180}{\pi} \times \text{rad} \tag{26.2}$$

Subunits of the Degree

For small angles, a degree may be subdivided into 60 *minutes* ($'$), and a minute into 60 *seconds* ($''$). Thus a minute is 1/60 degree, and a second is 1/3600 degree.[1] Angles smaller than 1 second are sometimes expressed as *milli-arcseconds* (1/1000 arcsecond).[2]

Solid Angle

A *solid angle* is the three-dimensional version of a plane angle, and is subtended by the vertex of a cone. The SI unit of solid angle is the *steradian* (sr), which is defined to be the solid angle that subtends an area equal to the square of the radius of a circle. By this definition, a full sphere subtends an area equal to the area of a sphere ($4\pi r^2$) divided by the square of its radius (r^2) — so a full sphere is 4π steradians, and a hemisphere is 2π steradians.

---

[1] Sometimes these units are called the *minute of arc* or *arcminute*, and the *second of arc* or *arcsecond* to distinguish them from the units of time that have the same name.

[2] In an old system (Ref. [4]), the second was further subdivided into 60 *thirds* ($'''$), the third into 60 *fourths* ($''''$), etc. Under this system, 1 milli-arcsecond is 3.6 fourths of arc. This system is no longer used, though; today the second of arc is simply subdivided into decimals (e.g. $32.86473''$).

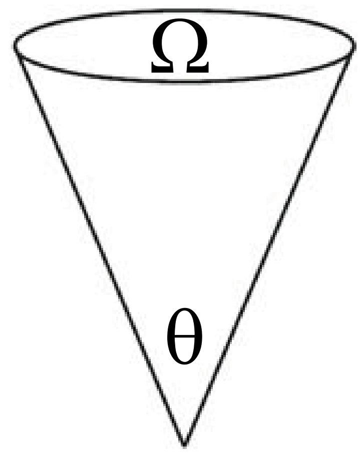


Figure 26.1: Relation between plane angle θ and solid angle Ω for a right circular cone.

There is a simple relation between plane angle and solid angle for a right circular cone. If the vertex of the cone subtends an angle θ (the *aperture angle* of the cone), then the corresponding solid angle Ω is (Fig. 26.1)

$$\Omega = 2\pi \left(1 - \cos\frac{\theta}{2}\right). \tag{26.3}$$

Another unit of solid angle is the *square degree* (deg^2):

$$\text{sq. deg.} = \text{sr} \times \left(\frac{180}{\pi}\right)^2. \tag{26.4}$$

In these units, a hemisphere is 20,626.48 deg^2, and a complete sphere is 41,252.96 deg^2.

Appendix 27

Newton's Laws of Motion (Latin)

Newton's laws of motion appear at the beginning of Book I of *Philosophiæ Naturalis Principia Mathematica*:

Axiomata, sive Leges Motus[1]

I. Corpus omne perseverare in statis suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

II. Mutationem motus proportionalem esse vi motrici impressæ, & fieri secundum lineam rectam qua vis illa imprimitur.

III. Actioni contrariam semper & æqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse æquales & in partes contrarias dirigi.

In modern language,

- *Vis* means *force*.
- *Actio* (action) and *reactio* (reaction) also refer to force.
- *Motus* (motion) is equivalent to what we now call *momentum*.

[1] Axioms, or Laws of Motion

I. Every body preserves in its state of being at rest or of moving uniformly straight forward, except in so far as it is compelled to change its state by forces impressed.

II. A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed.

III. To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and always opposite in direction.

Appendix 28

Round-Number Handbook of Physics

The one-page *Round Number Handbook of Physics* on the following page is by Edward M. Purcell of Harvard University, and appeared in the January 1983 issue of the *American Journal of Physics*. It is intended as a brief reference for doing quick "back of the envelope", order-of-magnitude calculations.

ROUND-NUMBER HANDBOOK OF PHYSICS

CONSTANTS

$c = 3\times10^{10}$ cm s^{-1}
$\hbar = 10^{-27}$ erg s
$N_0 = 6\times10^{23}$ mole^{-1}
$n_0 = 3\times10^{19}$ cm^{-3}
$g = 10^3$ cm s^{-2}
$e = 4.8\times10^{-10}$ esu
$= 1.6\times10^{-19}$ C
$k = 1.4\times10^{-16}$ erg deg^{-1}
$\alpha = e^2/\hbar c = 1/137$
$(\mu_0/\epsilon_0)^{1/2} = 377\Omega$
$G = 7\times10^{-8}$ g cm^{-4} s^{-2}
$\mu_0 = 4\pi\times10^{-7}$ N A^{-2}
$\epsilon_0 = 8.8\times10^{-12}$ N^{-1} A^2 m^{-2} s^2
$R = 2$ cal/mole deg

CONVERSIONS

1 cal = 4 J = 4×10^7 erg
1 N = 10^5 dyn
680 lumens = 1 W (5550 Å)
1 ft = 30 cm
1 lb = 4.4 N
1 ci = 4×10^{10} disint/s
1 eV = 1.6×10^{-12} erg
1 $\Omega^{-1} = 9\times10^{11}$ cm/s
pc(eV) = 300 Br(G cm)

MASSES

$m_e = 10^{-27}$ g
$m_{\text{pion}} = 270m_e$
$m_{\text{kaon}} = 1000m_e$
$m_{\text{nucleon}} = 2000m_e$
$m_e c^2 = 0.5$ MeV
$m_{\text{muon}} = 200m_e$

USEFUL NUMBERS

classical electron radius = $r_0 = e^2/m_e c^2 = 3\times10^{-13}$ cm
Bohr radius = $a_0 = \hbar^2/m_e e^2 = 5\times10^{-9}$ cm
Rydberg wavelength = $\lambda\!\!\!^{-}_R = \hbar^3 c/m_e e^4 = 7\times10^{-7}$cm
Compton wavelength = $\lambda\!\!\!^{-}_c = \hbar/m_e c = 4\times10^{-11}$ cm
Bohr magneton = $e\hbar/2mc = 10^{-20}$ erg/G
Stefan–Boltzman const = 6×10^{-12} W/deg^4 cm^2
Min. ionization loss: 2 MeV/g cm^2
$kT_{\text{room}} = 0.025$ eV
$R_{\text{nuclear}} = A^{1/3}\times10^{-13}$ cm
$e^2/a_0 = 26$ eV

$h\nu$(visible) = 2 eV
Band gaps: Si = 1.1 eV; Ge = 0.7 eV
Spin precession: e:3 MHz/G; p:4 kHz/G

MATERIALS

Resistivities in Ω cm: Cu:2×10^{-6} (room temp.)
H_2O(pure):2×10^7; seawater:25 Ω cm
Specific heat (solid or liquid) = 0.5 cal/cm^3 deg
Linear expansion (solid or liquid) = 2×10^{-5}/deg
Heat conduction (insulator) = 10^{-2}cal/s cm deg
(metal) = $1.0(\rho_{\text{Cu}}/\rho_{\text{metal}})$cal/s cm deg
Heat of combustion (food or fuel) = 10^4 cal/g
Heat of vaporization = 10^4 cal/mole
Elastic moduli (solids) = 10^{11}–10^{12} dyn/cm^2
Tensile strength (solids) = 10^8–10^{10} dyn/cm^2
Surface tension: H_2O = 50 dyn/cm
Diffusion: H_2O 10^{-5}, air:0.2 cm^2/s
Viscosity: H_2O 10^{-2}, air:2×10^{-4} dyn s/cm^2

ASTRONOMICAL

1 pc = 3×10^{18} cm
1 mag = -4 dB
$m_{\text{abs}} = m$ at 10 pc
m_{abs}(sun) = $+5$
B_{Earth}(pole) = 0.5 G
$M_{\text{Earth}} = 6\times10^{27}$ g
$R_{\text{Earth}} = 6\times10^8$ cm
$M_\odot = 2\times10^{33}$ g
$R_\odot = 8\times10^{10}$ cm
$L_\odot = 2\times10^{33}$ erg/s = 1 kW/m^2 at Earth
$r_{\text{moon}} = 4\times10^{10}$ cm
$r_{\text{sun}} = 1$ AU = 1.5×10^{13} cm
$M_{\text{Galaxy}} = 2\times10^{44}$ g
Distance to center of galaxy = 3×10^{22} cm
Distance between galaxies = 10^{25} cm
Energy density: starlight = 10^{-12} erg/cm^3
Primary cosmic rays: 1/cm^2 s
$R_{\text{Universe}} = 3000$ Mpc

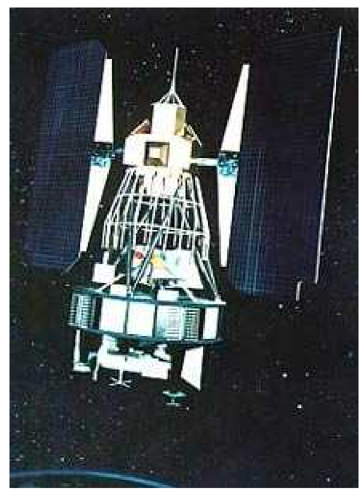
ATMOSPHERE (STP)

$P_{\text{atm}} = 10^6$ dyn/cm^2 = 15 psi
$V_{\text{sound}} = V_{\text{molec}} = 4\times10^4$ cm/s
Radiation length = 36 g/cm^2
Density = 10^{-3} g/cm^3
Mean free path = 7×10^{-6} cm
Scale height = 8 km

Appendix 29

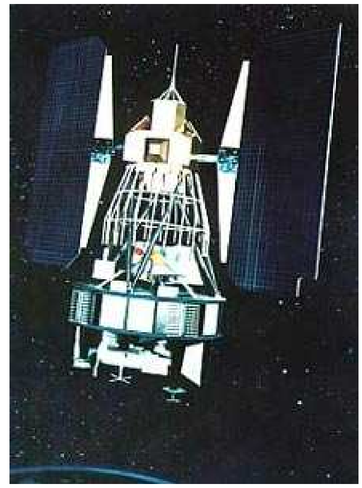
The Landsat Missions

Landsat 1



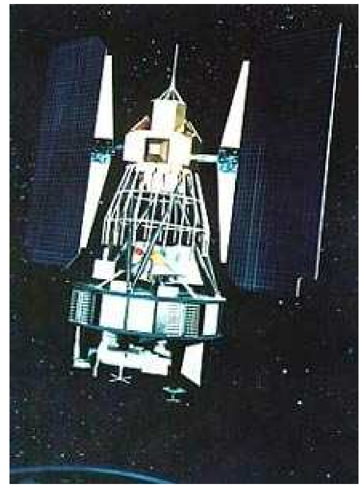
- Launch: July 23, 1972
- End of mission: January 6, 1978
- Instruments: Return Beacon Vidicon (RBV); Multi-Spectral Scanner (MSS).
- Notes: Originally named Earth Resources Technology Satellite 1 (ERTS-1).

Landsat 2



- Launch: January 22, 1975
- End of mission: February 25, 1982
- Instruments: Return Beacon Vidicon (RBV); Multi-Spectral Scanner (MSS).
- Notes: Nearly identical to Landsat 1.

Landsat 3



- Launch: March 5, 1978
- End of mission: March 31, 1983
- Instruments: Return Beacon Vidicon (RBV); Multi-Spectral Scanner (MSS).
- Nearly identical to Landsats 1 and 2.

Landsat 4

- Launch: July 16, 1982
- End of mission: December 14, 1993
- Instruments: Multi-Spectral Scanner (MSS); Thematic Mapper.

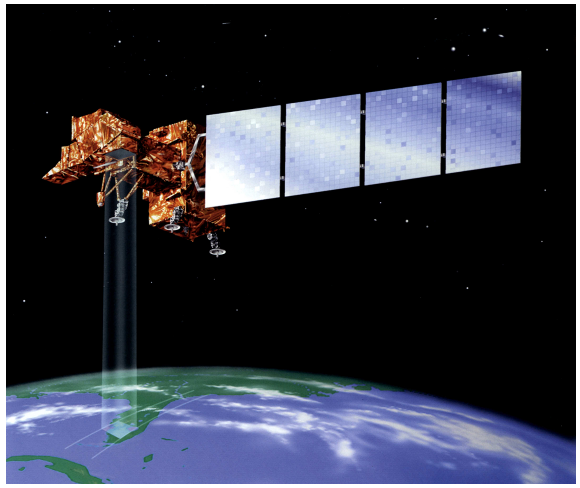
Landsat 5

- Launch: March 1, 1984
- End of mission: June 5, 2013
- Instruments: Multi-Spectral Scanner (MSS); Thematic Mapper.
- Notes: Nearly identical to Landsat 4.

Landsat 6

- Launch: October 5, 1993
- End of mission: October 5, 1993
- Instruments: Multi-Spectral Scanner (MSS); Enhanced Thematic Mapper.
- Notes: Failed to reach orbit.

Landsat 7



- Launch: April 15, 1999
- End of mission: April 6, 2022
- Instruments: Enhanced Thematic Mapper (ETM+).

Landsat 8

- Launch: February 11, 2013
- End of mission: Still active
- Instruments: Operational Land Imager (OLI); Thermal Infrared Sensor (TIS)

Landsat 9

- Launch: September 27, 2021
- End of mission: Still active
- Instruments: Operational Land Imager (OLI); Thermal Infrared Sensor (TIS)
- Notes: Nearly identical to Landsat 8.

Appendix 30

Fundamental Physical Constants — Extensive Listing

The following tables, published by the National Institutes of Science and Technology (NIST), give the current best estimates of a large number of fundamental physical constants. These values were determined by the Committee on Data for Science and Technology (CODATA) for 2018, and are a best fit of the constants to the latest experimental results. These values include the 2019 re-definition of SI units.

(Source: `https://physics.nist.gov/cuu/Constants/index.html`)

Fundamental Physical Constants — Extensive Listing

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
		UNIVERSAL		
speed of light in vacuum	c	$299\,792\,458$	m s^{-1}	exact
vacuum magnetic permeability $4\pi\alpha\hbar/e^2c$	μ_0	$1.256\,637\,062\,12(19) \times 10^{-6}$	N A^{-2}	1.5×10^{-10}
$\mu_0/(4\pi \times 10^{-7})$		$1.000\,000\,000\,55(15)$	N A^{-2}	1.5×10^{-10}
vacuum electric permittivity $1/\mu_0 c^2$	ϵ_0	$8.854\,187\,8128(13) \times 10^{-12}$	F m^{-1}	1.5×10^{-10}
characteristic impedance of vacuum $\mu_0 c$	Z_0	$376.730\,313\,668(57)$	Ω	1.5×10^{-10}
Newtonian constant of gravitation	G	$6.674\,30(15) \times 10^{-11}$	m^3 kg^{-1} s^{-2}	2.2×10^{-5}
	$G/\hbar c$	$6.708\,83(15) \times 10^{-39}$	(GeV/c^2)$^{-2}$	2.2×10^{-5}
Planck constant*	h	$6.626\,070\,15 \times 10^{-34}$	J Hz^{-1}	exact
		$4.135\,667\,696 \ldots \times 10^{-15}$	eV Hz^{-1}	exact
	$\hbar$	$1.054\,571\,817 \ldots \times 10^{-34}$	J s	exact
		$6.582\,119\,569 \ldots \times 10^{-16}$	eV s	exact
	$\hbar c$	$197.326\,980\,4 \ldots$	MeV fm	exact
Planck mass $(\hbar c/G)^{1/2}$	m_P	$2.176\,434(24) \times 10^{-8}$	kg	1.1×10^{-5}
energy equivalent	$m_P c^2$	$1.220\,890(14) \times 10^{19}$	GeV	1.1×10^{-5}
Planck temperature $(\hbar c^5/G)^{1/2}/k$	T_P	$1.416\,784(16) \times 10^{32}$	K	1.1×10^{-5}
Planck length $\hbar/m_P c = (\hbar G/c^3)^{1/2}$	l_P	$1.616\,255(18) \times 10^{-35}$	m	1.1×10^{-5}
Planck time $l_P/c = (\hbar G/c^5)^{1/2}$	t_P	$5.391\,247(60) \times 10^{-44}$	s	1.1×10^{-5}
		ELECTROMAGNETIC		
elementary charge	e	$1.602\,176\,634 \times 10^{-19}$	C	exact
	$e/\hbar$	$1.519\,267\,447 \ldots \times 10^{15}$	A J^{-1}	exact
magnetic flux quantum $2\pi\hbar/(2e)$	Φ_0	$2.067\,833\,848 \ldots \times 10^{-15}$	Wb	exact
conductance quantum $2e^2/2\pi\hbar$	G_0	$7.748\,091\,729 \ldots \times 10^{-5}$	S	exact
inverse of conductance quantum	G_0^{-1}	$12\,906.403\,72 \ldots$	Ω	exact
Josephson constant $2e/h$	K_J	$483\,597.848\,4 \ldots \times 10^9$	Hz V^{-1}	exact
von Klitzing constant $\mu_0 c/2\alpha = 2\pi\hbar/e^2$	R_K	$25\,812.807\,45 \ldots$	Ω	exact
Bohr magneton $e\hbar/2m_e$	μ_B	$9.274\,010\,0783(28) \times 10^{-24}$	J T^{-1}	3.0×10^{-10}
		$5.788\,381\,8060(17) \times 10^{-5}$	eV T^{-1}	3.0×10^{-10}
	μ_B/h	$1.399\,624\,493\,61(42) \times 10^{10}$	Hz T^{-1}	3.0×10^{-10}
	μ_B/hc	$46.686\,447\,783(14)$	[m^{-1} T^{-1}]†	3.0×10^{-10}
	μ_B/k	$0.671\,713\,815\,63(20)$	K T^{-1}	3.0×10^{-10}
nuclear magneton $e\hbar/2m_p$	μ_N	$5.050\,783\,7461(15) \times 10^{-27}$	J T^{-1}	3.1×10^{-10}
		$3.152\,451\,258\,44(96) \times 10^{-8}$	eV T^{-1}	3.1×10^{-10}
	μ_N/h	$7.622\,593\,2291(23)$	MHz T^{-1}	3.1×10^{-10}
	μ_N/hc	$2.542\,623\,413\,53(78) \times 10^{-2}$	[m^{-1} T^{-1}]†	3.1×10^{-10}
	μ_N/k	$3.658\,267\,7756(11) \times 10^{-4}$	K T^{-1}	3.1×10^{-10}
		ATOMIC AND NUCLEAR		
		General		
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,5693(11) \times 10^{-3}$		1.5×10^{-10}
inverse fine-structure constant	α^{-1}	$137.035\,999\,084(21)$		1.5×10^{-10}
Rydberg frequency $\alpha^2 m_e c^2/2h = E_h/2h$	cR_∞	$3.289\,841\,960\,2508(64) \times 10^{15}$	Hz	1.9×10^{-12}
energy equivalent	$hc\,R_\infty$	$2.179\,872\,361\,1035(42) \times 10^{-18}$	J	1.9×10^{-12}
		$13.605\,693\,122\,994(26)$	eV	1.9×10^{-12}
Rydberg constant	R_∞	$10\,973\,731.568\,160(21)$	[m^{-1}]†	1.9×10^{-12}
Bohr radius $\hbar/\alpha m_e c = 4\pi\epsilon_0\hbar^2/m_e e^2$	a_0	$5.291\,772\,109\,03(80) \times 10^{-11}$	m	1.5×10^{-10}
Hartree energy $\alpha^2 m_e c^2 = e^2/4\pi\epsilon_0 a_0 = 2hcR_\infty$	E_h	$4.359\,744\,722\,2071(85) \times 10^{-18}$	J	1.9×10^{-12}
		$27.211\,386\,245\,988(53)$	eV	1.9×10^{-12}
quantum of circulation	$\pi\hbar/m_e$	$3.636\,947\,5516(11) \times 10^{-4}$	m^2 s^{-1}	3.0×10^{-10}

Fundamental Physical Constants — Extensive Listing

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
	$2\pi\hbar/m_e$	$7.273\,895\,1032(22)\times10^{-4}$	$m^2\ s^{-1}$	3.0×10^{-10}
		Electroweak		
Fermi coupling constant‡	$G_F/(\hbar c)^3$	$1.166\,3787(6)\times10^{-5}$	GeV^{-2}	5.1×10^{-7}
weak mixing angle§ θ_W (on-shell scheme)				
$\sin^2\theta_W = s_W^2 \equiv 1-(m_W/m_Z)^2$	$\sin^2\theta_W$	$0.222\,90(30)$		1.3×10^{-3}
		Electron, e^-		
electron mass	m_e	$9.109\,383\,7015(28)\times10^{-31}$	kg	3.0×10^{-10}
		$5.485\,799\,090\,65(16)\times10^{-4}$	u	2.9×10^{-11}
energy equivalent	m_ec^2	$8.187\,105\,7769(25)\times10^{-14}$	J	3.0×10^{-10}
		$0.510\,998\,950\,00(15)$	MeV	3.0×10^{-10}
electron-muon mass ratio	m_e/m_μ	$4.836\,331\,69(11)\times10^{-3}$		2.2×10^{-8}
electron-tau mass ratio	m_e/m_τ	$2.875\,85(19)\times10^{-4}$		6.8×10^{-5}
electron-proton mass ratio	m_e/m_p	$5.446\,170\,214\,87(33)\times10^{-4}$		6.0×10^{-11}
electron-neutron mass ratio	m_e/m_n	$5.438\,673\,4424(26)\times10^{-4}$		4.8×10^{-10}
electron-deuteron mass ratio	m_e/m_d	$2.724\,437\,107\,462(96)\times10^{-4}$		3.5×10^{-11}
electron-triton mass ratio	m_e/m_t	$1.819\,200\,062\,251(90)\times10^{-4}$		5.0×10^{-11}
electron-helion mass ratio	m_e/m_h	$1.819\,543\,074\,573(79)\times10^{-4}$		4.3×10^{-11}
electron to alpha particle mass ratio	m_e/m_α	$1.370\,933\,554\,787(45)\times10^{-4}$		3.3×10^{-11}
electron charge to mass quotient	$-e/m_e$	$-1.758\,820\,010\,76(53)\times10^{11}$	$C\ kg^{-1}$	3.0×10^{-10}
electron molar mass N_Am_e	$M(e), M_e$	$5.485\,799\,0888(17)\times10^{-7}$	$kg\ mol^{-1}$	3.0×10^{-10}
reduced Compton wavelength $\hbar/m_ec=\alpha a_0$	$\lambda\!\!\!^{-}_C$	$3.861\,592\,6796(12)\times10^{-13}$	m	3.0×10^{-10}
Compton wavelength	λ_C	$2.426\,310\,238\,67(73)\times10^{-12}$	[m]†	3.0×10^{-10}
classical electron radius α^2a_0	r_e	$2.817\,940\,3262(13)\times10^{-15}$	m	4.5×10^{-10}
Thomson cross section $(8\pi/3)r_e^2$	σ_e	$6.652\,458\,7321(60)\times10^{-29}$	m^2	9.1×10^{-10}
electron magnetic moment	μ_e	$-9.284\,764\,7043(28)\times10^{-24}$	$J\ T^{-1}$	3.0×10^{-10}
to Bohr magneton ratio	μ_e/μ_B	$-1.001\,159\,652\,181\,28(18)$		1.7×10^{-13}
to nuclear magneton ratio	μ_e/μ_N	$-1838.281\,971\,88(11)$		6.0×10^{-11}
electron magnetic moment				
anomaly $\lvert\mu_e\rvert/\mu_B-1$	a_e	$1.159\,652\,181\,28(18)\times10^{-3}$		1.5×10^{-10}
electron g-factor $-2(1+a_e)$	g_e	$-2.002\,319\,304\,362\,56(35)$		1.7×10^{-13}
electron-muon magnetic moment ratio	μ_e/μ_μ	$206.766\,9883(46)$		2.2×10^{-8}
electron-proton magnetic moment ratio	μ_e/μ_p	$-658.210\,687\,89(20)$		3.0×10^{-10}
electron to shielded proton magnetic				
moment ratio (H_2O, sphere, 25 °C)	μ_e/μ'_p	$-658.227\,5971(72)$		1.1×10^{-8}
electron-neutron magnetic moment ratio	μ_e/μ_n	$960.920\,50(23)$		2.4×10^{-7}
electron-deuteron magnetic moment ratio	μ_e/μ_d	$-2143.923\,4915(56)$		2.6×10^{-9}
electron to shielded helion magnetic				
moment ratio (gas, sphere, 25 °C)	μ_e/μ'_h	$864.058\,257(10)$		1.2×10^{-8}
electron gyromagnetic ratio $2\lvert\mu_e\rvert/\hbar$	γ_e	$1.760\,859\,630\,23(53)\times10^{11}$	$s^{-1}\ T^{-1}$	3.0×10^{-10}
		$28\,024.951\,4242(85)$	$MHz\ T^{-1}$	3.0×10^{-10}
		Muon, μ^-		
muon mass	m_μ	$1.883\,531\,627(42)\times10^{-28}$	kg	2.2×10^{-8}
		$0.113\,428\,9259(25)$	u	2.2×10^{-8}
energy equivalent	$m_\mu c^2$	$1.692\,833\,804(38)\times10^{-11}$	J	2.2×10^{-8}
		$105.658\,3755(23)$	MeV	2.2×10^{-8}
muon-electron mass ratio	m_μ/m_e	$206.768\,2830(46)$		2.2×10^{-8}
muon-tau mass ratio	m_μ/m_τ	$5.946\,35(40)\times10^{-2}$		6.8×10^{-5}
muon-proton mass ratio	m_μ/m_p	$0.112\,609\,5264(25)$		2.2×10^{-8}

Fundamental Physical Constants — Extensive Listing

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
muon-neutron mass ratio	m_μ/m_n	$0.112\,454\,5170(25)$		2.2×10^{-8}
muon molar mass $N_A m_\mu$	$M(\mu), M_\mu$	$1.134\,289\,259(25)\times10^{-4}$	kg mol^{-1}	2.2×10^{-8}
reduced muon Compton wavelength $\hbar/m_\mu c$	$\lambda\!\!\!^{-}_{C,\mu}$	$1.867\,594\,306(42)\times10^{-15}$	m	2.2×10^{-8}
muon Compton wavelength	$\lambda_{C,\mu}$	$1.173\,444\,110(26)\times10^{-14}$	[m]†	2.2×10^{-8}
muon magnetic moment	μ_μ	$-4.490\,448\,30(10)\times10^{-26}$	J T^{-1}	2.2×10^{-8}
to Bohr magneton ratio	μ_μ/μ_B	$-4.841\,970\,47(11)\times10^{-3}$		2.2×10^{-8}
to nuclear magneton ratio	μ_μ/μ_N	$-8.890\,597\,03(20)$		2.2×10^{-8}
muon magnetic moment anomaly $\lvert\mu_\mu\rvert/(e\hbar/2m_\mu)-1$	a_μ	$1.165\,920\,89(63)\times10^{-3}$		5.4×10^{-7}
muon g-factor $-2(1+a_\mu)$	g_μ	$-2.002\,331\,8418(13)$		6.3×10^{-10}
muon-proton magnetic moment ratio	μ_μ/μ_p	$-3.183\,345\,142(71)$		2.2×10^{-8}
		Tau, τ^-		
tau mass¶	m_τ	$3.167\,54(21)\times10^{-27}$	kg	6.8×10^{-5}
		$1.907\,54(13)$	u	6.8×10^{-5}
energy equivalent	$m_\tau c^2$	$2.846\,84(19)\times10^{-10}$	J	6.8×10^{-5}
		$1776.86(12)$	MeV	6.8×10^{-5}
tau-electron mass ratio	m_τ/m_e	$3477.23(23)$		6.8×10^{-5}
tau-muon mass ratio	m_τ/m_μ	$16.8170(11)$		6.8×10^{-5}
tau-proton mass ratio	m_τ/m_p	$1.893\,76(13)$		6.8×10^{-5}
tau-neutron mass ratio	m_τ/m_n	$1.891\,15(13)$		6.8×10^{-5}
tau molar mass $N_A m_\tau$	$M(\tau), M_\tau$	$1.907\,54(13)\times10^{-3}$	kg mol^{-1}	6.8×10^{-5}
reduced tau Compton wavelength $\hbar/m_\tau c$	$\lambda\!\!\!^{-}_{C,\tau}$	$1.110\,538(75)\times10^{-16}$	m	6.8×10^{-5}
tau Compton wavelength	$\lambda_{C,\tau}$	$6.977\,71(47)\times10^{-16}$	[m]†	6.8×10^{-5}
		Proton, p		
proton mass	m_p	$1.672\,621\,923\,69(51)\times10^{-27}$	kg	3.1×10^{-10}
		$1.007\,276\,466\,621(53)$	u	5.3×10^{-11}
energy equivalent	$m_p c^2$	$1.503\,277\,615\,98(46)\times10^{-10}$	J	3.1×10^{-10}
		$938.272\,088\,16(29)$	MeV	3.1×10^{-10}
proton-electron mass ratio	m_p/m_e	$1836.152\,673\,43(11)$		6.0×10^{-11}
proton-muon mass ratio	m_p/m_μ	$8.880\,243\,37(20)$		2.2×10^{-8}
proton-tau mass ratio	m_p/m_τ	$0.528\,051(36)$		6.8×10^{-5}
proton-neutron mass ratio	m_p/m_n	$0.998\,623\,478\,12(49)$		4.9×10^{-10}
proton charge to mass quotient	e/m_p	$9.578\,833\,1560(29)\times10^{7}$	C kg^{-1}	3.1×10^{-10}
proton molar mass $N_A m_p$	$M(\text{p}), M_p$	$1.007\,276\,466\,27(31)\times10^{-3}$	kg mol^{-1}	3.1×10^{-10}
reduced proton Compton wavelength $\hbar/m_p c$	$\lambda\!\!\!^{-}_{C,p}$	$2.103\,089\,103\,36(64)\times10^{-16}$	m	3.1×10^{-10}
proton Compton wavelength	$\lambda_{C,p}$	$1.321\,409\,855\,39(40)\times10^{-15}$	[m]†	3.1×10^{-10}
proton rms charge radius	r_p	$8.414(19)\times10^{-16}$	m	2.2×10^{-3}
proton magnetic moment	μ_p	$1.410\,606\,797\,36(60)\times10^{-26}$	J T^{-1}	4.2×10^{-10}
to Bohr magneton ratio	μ_p/μ_B	$1.521\,032\,202\,30(46)\times10^{-3}$		3.0×10^{-10}
to nuclear magneton ratio	μ_p/μ_N	$2.792\,847\,344\,63(82)$		2.9×10^{-10}
proton g-factor $2\mu_p/\mu_N$	g_p	$5.585\,694\,6893(16)$		2.9×10^{-10}
proton-neutron magnetic moment ratio	μ_p/μ_n	$-1.459\,898\,05(34)$		2.4×10^{-7}
shielded proton magnetic moment (H_2O, sphere, 25 °C)	μ'_p	$1.410\,570\,560(15)\times10^{-26}$	J T^{-1}	1.1×10^{-8}
to Bohr magneton ratio	μ'_p/μ_B	$1.520\,993\,128(17)\times10^{-3}$		1.1×10^{-8}
to nuclear magneton ratio	μ'_p/μ_N	$2.792\,775\,599(30)$		1.1×10^{-8}
proton magnetic shielding correction $1-\mu'_p/\mu_p$ (H_2O, sphere, 25 °C)	σ'_p	$2.5689(11)\times10^{-5}$		4.2×10^{-4}

Fundamental Physical Constants — Extensive Listing

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
proton gyromagnetic ratio $2\mu_p/\hbar$	γ_p	$2.675\,221\,8744(11)\times10^{8}$	$s^{-1}\,T^{-1}$	4.2×10^{-10}
		$42.577\,478\,518(18)$	MHz T^{-1}	4.2×10^{-10}
shielded proton gyromagnetic ratio				
$2\mu'_p/\hbar$ (H_2O, sphere, 25 °C)	γ'_p	$2.675\,153\,151(29)\times10^{8}$	$s^{-1}\,T^{-1}$	1.1×10^{-8}
		$42.576\,384\,74(46)$	MHz T^{-1}	1.1×10^{-8}
		Neutron, n		
neutron mass	m_n	$1.674\,927\,498\,04(95)\times10^{-27}$	kg	5.7×10^{-10}
		$1.008\,664\,915\,95(49)$	u	4.8×10^{-10}
energy equivalent	m_nc^2	$1.505\,349\,762\,87(86)\times10^{-10}$	J	5.7×10^{-10}
		$939.565\,420\,52(54)$	MeV	5.7×10^{-10}
neutron-electron mass ratio	m_n/m_e	$1838.683\,661\,73(89)$		4.8×10^{-10}
neutron-muon mass ratio	m_n/m_μ	$8.892\,484\,06(20)$		2.2×10^{-8}
neutron-tau mass ratio	m_n/m_τ	$0.528\,779(36)$		6.8×10^{-5}
neutron-proton mass ratio	m_n/m_p	$1.001\,378\,419\,31(49)$		4.9×10^{-10}
neutron-proton mass difference	m_n-m_p	$2.305\,574\,35(82)\times10^{-30}$	kg	3.5×10^{-7}
		$1.388\,449\,33(49)\times10^{-3}$	u	3.5×10^{-7}
energy equivalent	$(m_n-m_p)c^2$	$2.072\,146\,89(74)\times10^{-13}$	J	3.5×10^{-7}
		$1.293\,332\,36(46)$	MeV	3.5×10^{-7}
neutron molar mass N_Am_n	$M(n), M_n$	$1.008\,664\,915\,60(57)\times10^{-3}$	kg mol^{-1}	5.7×10^{-10}
reduced neutron Compton wavelength $\hbar/m_nc$	$\lambda\!\!\!^{-}_{C,n}$	$2.100\,194\,1552(12)\times10^{-16}$	m	5.7×10^{-10}
neutron Compton wavelength	$\lambda_{C,n}$	$1.319\,590\,905\,81(75)\times10^{-15}$	[m]†	5.7×10^{-10}
neutron magnetic moment	μ_n	$-9.662\,3651(23)\times10^{-27}$	J T^{-1}	2.4×10^{-7}
to Bohr magneton ratio	μ_n/μ_B	$-1.041\,875\,63(25)\times10^{-3}$		2.4×10^{-7}
to nuclear magneton ratio	μ_n/μ_N	$-1.913\,042\,73(45)$		2.4×10^{-7}
neutron g-factor $2\mu_n/\mu_N$	g_n	$-3.826\,085\,45(90)$		2.4×10^{-7}
neutron-electron magnetic moment ratio	μ_n/μ_e	$1.040\,668\,82(25)\times10^{-3}$		2.4×10^{-7}
neutron-proton magnetic moment ratio	μ_n/μ_p	$-0.684\,979\,34(16)$		2.4×10^{-7}
neutron to shielded proton magnetic				
moment ratio (H_2O, sphere, 25 °C)	μ_n/μ'_p	$-0.684\,996\,94(16)$		2.4×10^{-7}
neutron gyromagnetic ratio $2\lvert\mu_n\rvert/\hbar$	γ_n	$1.832\,471\,71(43)\times10^{8}$	$s^{-1}\,T^{-1}$	2.4×10^{-7}
		$29.164\,6931(69)$	MHz T^{-1}	2.4×10^{-7}
		Deuteron, d		
deuteron mass	m_d	$3.343\,583\,7724(10)\times10^{-27}$	kg	3.0×10^{-10}
		$2.013\,553\,212\,745(40)$	u	2.0×10^{-11}
energy equivalent	m_dc^2	$3.005\,063\,231\,02(91)\times10^{-10}$	J	3.0×10^{-10}
		$1875.612\,942\,57(57)$	MeV	3.0×10^{-10}
deuteron-electron mass ratio	m_d/m_e	$3670.482\,967\,88(13)$		3.5×10^{-11}
deuteron-proton mass ratio	m_d/m_p	$1.999\,007\,501\,39(11)$		5.6×10^{-11}
deuteron molar mass N_Am_d	$M(d), M_d$	$2.013\,553\,212\,05(61)\times10^{-3}$	kg mol^{-1}	3.0×10^{-10}
deuteron rms charge radius	r_d	$2.127\,99(74)\times10^{-15}$	m	3.5×10^{-4}
deuteron magnetic moment	μ_d	$4.330\,735\,094(11)\times10^{-27}$	J T^{-1}	2.6×10^{-9}
to Bohr magneton ratio	μ_d/μ_B	$4.669\,754\,570(12)\times10^{-4}$		2.6×10^{-9}
to nuclear magneton ratio	μ_d/μ_N	$0.857\,438\,2338(22)$		2.6×10^{-9}
deuteron g-factor μ_d/μ_N	g_d	$0.857\,438\,2338(22)$		2.6×10^{-9}
deuteron-electron magnetic moment ratio	μ_d/μ_e	$-4.664\,345\,551(12)\times10^{-4}$		2.6×10^{-9}
deuteron-proton magnetic moment ratio	μ_d/μ_p	$0.307\,012\,209\,39(79)$		2.6×10^{-9}
deuteron-neutron magnetic moment ratio	μ_d/μ_n	$-0.448\,206\,53(11)$		2.4×10^{-7}

Fundamental Physical Constants — Extensive Listing

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
		Triton, t		
triton mass	m_t	$5.007\,356\,7446(15) \times 10^{-27}$	kg	3.0×10^{-10}
		$3.015\,500\,716\,21(12)$	u	4.0×10^{-11}
energy equivalent	$m_t c^2$	$4.500\,387\,8060(14) \times 10^{-10}$	J	3.0×10^{-10}
		$2808.921\,132\,98(85)$	MeV	3.0×10^{-10}
triton-electron mass ratio	m_t/m_e	$5496.921\,535\,73(27)$		5.0×10^{-11}
triton-proton mass ratio	m_t/m_p	$2.993\,717\,034\,14(15)$		5.0×10^{-11}
triton molar mass $N_A m_t$	$M(t), M_t$	$3.015\,500\,715\,17(92) \times 10^{-3}$	kg mol^{-1}	3.0×10^{-10}
triton magnetic moment	μ_t	$1.504\,609\,5202(30) \times 10^{-26}$	J T^{-1}	2.0×10^{-9}
to Bohr magneton ratio	μ_t/μ_B	$1.622\,393\,6651(32) \times 10^{-3}$		2.0×10^{-9}
to nuclear magneton ratio	μ_t/μ_N	$2.978\,962\,4656(59)$		2.0×10^{-9}
triton g-factor $2\mu_t/\mu_N$	g_t	$5.957\,924\,931(12)$		2.0×10^{-9}
		Helion, h		
helion mass	m_h	$5.006\,412\,7796(15) \times 10^{-27}$	kg	3.0×10^{-10}
		$3.014\,932\,247\,175(97)$	u	3.2×10^{-11}
energy equivalent	$m_h c^2$	$4.499\,539\,4125(14) \times 10^{-10}$	J	3.0×10^{-10}
		$2808.391\,607\,43(85)$	MeV	3.0×10^{-10}
helion-electron mass ratio	m_h/m_e	$5495.885\,280\,07(24)$		4.3×10^{-11}
helion-proton mass ratio	m_h/m_p	$2.993\,152\,671\,67(13)$		4.4×10^{-11}
helion molar mass $N_A m_h$	$M(h), M_h$	$3.014\,932\,246\,13(91) \times 10^{-3}$	kg mol^{-1}	3.0×10^{-10}
helion magnetic moment	μ_h	$-1.074\,617\,532(13) \times 10^{-26}$	J T^{-1}	1.2×10^{-8}
to Bohr magneton ratio	μ_h/μ_B	$-1.158\,740\,958(14) \times 10^{-3}$		1.2×10^{-8}
to nuclear magneton ratio	μ_h/μ_N	$-2.127\,625\,307(25)$		1.2×10^{-8}
helion g-factor $2\mu_h/\mu_N$	g_h	$-4.255\,250\,615(50)$		1.2×10^{-8}
shielded helion magnetic moment (gas, sphere, 25 °C)	μ'_h	$-1.074\,553\,090(13) \times 10^{-26}$	J T^{-1}	1.2×10^{-8}
to Bohr magneton ratio	μ'_h/μ_B	$-1.158\,671\,471(14) \times 10^{-3}$		1.2×10^{-8}
to nuclear magneton ratio	μ'_h/μ_N	$-2.127\,497\,719(25)$		1.2×10^{-8}
shielded helion to proton magnetic moment ratio (gas, sphere, 25 °C)	μ'_h/μ_p	$-0.761\,766\,5618(89)$		1.2×10^{-8}
shielded helion to shielded proton magnetic moment ratio (gas/H_2O, spheres, 25 °C)	μ'_h/μ'_p	$-0.761\,786\,1313(33)$		4.3×10^{-9}
shielded helion gyromagnetic ratio $2\|\mu'_h\|/\hbar$ (gas, sphere, 25 °C)	γ'_h	$2.037\,894\,569(24) \times 10^{8}$	s^{-1} T^{-1}	1.2×10^{-8}
		$32.434\,099\,42(38)$	MHz T^{-1}	1.2×10^{-8}
		Alpha particle, α		
alpha particle mass	m_α	$6.644\,657\,3357(20) \times 10^{-27}$	kg	3.0×10^{-10}
		$4.001\,506\,179\,127(63)$	u	1.6×10^{-11}
energy equivalent	$m_\alpha c^2$	$5.971\,920\,1914(18) \times 10^{-10}$	J	3.0×10^{-10}
		$3727.379\,4066(11)$	MeV	3.0×10^{-10}
alpha particle to electron mass ratio	m_α/m_e	$7294.299\,541\,42(24)$		3.3×10^{-11}
alpha particle to proton mass ratio	m_α/m_p	$3.972\,599\,690\,09(22)$		5.5×10^{-11}
alpha particle molar mass $N_A m_\alpha$	$M(\alpha), M_\alpha$	$4.001\,506\,1777(12) \times 10^{-3}$	kg mol^{-1}	3.0×10^{-10}
		PHYSICOCHEMICAL		
Avogadro constant	N_A	$6.022\,140\,76 \times 10^{23}$	mol^{-1}	exact
Boltzmann constant	k	$1.380\,649 \times 10^{-23}$	J K^{-1}	exact
		$8.617\,333\,262\ldots \times 10^{-5}$	eV K^{-1}	exact
	k/h	$2.083\,661\,912\ldots \times 10^{10}$	Hz K^{-1}	exact

Fundamental Physical Constants — Extensive Listing

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
	k/hc	69.503 480 04 . . .	$[\mathrm{m^{-1}\ K^{-1}}]^\dagger$	exact
atomic mass constant‖				
$m_u = \frac{1}{12}m(^{12}\mathrm{C}) = 2hc\,R_\infty/\alpha^2c^2A_r(\mathrm{e})$	m_u	$1.660\,539\,066\,60(50) \times 10^{-27}$	kg	3.0×10^{-10}
energy equivalent	m_uc^2	$1.492\,418\,085\,60(45) \times 10^{-10}$	J	3.0×10^{-10}
		931.494 102 42(28)	MeV	3.0×10^{-10}
molar mass constant‖	M_u	$0.999\,999\,999\,65(30) \times 10^{-3}$	kg mol^{-1}	3.0×10^{-10}
molar mass‖ of carbon-12 $A_r(^{12}\mathrm{C})M_u$	$M(^{12}\mathrm{C})$	$11.999\,999\,9958(36) \times 10^{-3}$	kg mol^{-1}	3.0×10^{-10}
molar Planck constant	N_Ah	$3.990\,312\,712\ldots \times 10^{-10}$	J Hz^{-1} mol^{-1}	exact
molar gas constant N_Ak	R	8.314 462 618 . . .	J mol^{-1} K^{-1}	exact
Faraday constant N_Ae	F	96 485.332 12 . . .	C mol^{-1}	exact
standard-state pressure		100 000	Pa	exact
standard atmosphere		101 325	Pa	exact
molar volume of ideal gas RT/p				
$T = 273.15$ K, $p = 100$ kPa or standard-state pressure	V_m	$22.710\,954\,64\ldots \times 10^{-3}$	m^3 mol^{-1}	exact
Loschmidt constant N_A/V_m	n_0	$2.651\,645\,804\ldots \times 10^{25}$	m^{-3}	exact
molar volume of ideal gas RT/p				
$T = 273.15$ K, $p = 101.325$ kPa or standard atmosphere	V_m	$22.413\,969\,54\ldots \times 10^{-3}$	m^3 mol^{-1}	exact
Loschmidt constant N_A/V_m	n_0	$2.686\,780\,111\ldots \times 10^{25}$	m^{-3}	exact
Sackur-Tetrode (absolute entropy) constant**				
$\frac{5}{2} + \ln[(m_ukT_1/2\pi\hbar^2)^{3/2}kT_1/p_0]$				
$T_1 = 1$ K, $p_0 = 100$ kPa or standard-state pressure	S_0/R	$-1.151\,707\,537\,06(45)$		3.9×10^{-10}
$T_1 = 1$ K, $p_0 = 101.325$ kPa or standard atmosphere		$-1.164\,870\,523\,58(45)$		3.9×10^{-10}
Stefan-Boltzmann constant				
$(\pi^2/60)k^4/\hbar^3c^2$	σ	$5.670\,374\,419\ldots \times 10^{-8}$	W m^{-2} K^{-4}	exact
first radiation constant for spectral				
radiance $2hc^2\,\mathrm{sr}^{-1}$	c_{1L}	$1.191\,042\,972\ldots \times 10^{-16}$	$[\mathrm{W\ m^2\ sr^{-1}}]^{\dagger\dagger}$	exact
first radiation constant $2\pi hc^2 = \pi\,\mathrm{sr}\,c_{1L}$	c_1	$3.741\,771\,852\ldots \times 10^{-16}$	$[\mathrm{W\ m^2}]^{\dagger\dagger}$	exact
second radiation constant hc/k	c_2	$1.438\,776\,877\ldots \times 10^{-2}$	$[\mathrm{m\ K}]^\dagger$	exact
Wien displacement law constants				
$b = \lambda_{max}T = c_2/4.965\,114\,231...$	b	$2.897\,771\,955\ldots \times 10^{-3}$	$[\mathrm{m\ K}]^\dagger$	exact
$b' = \nu_{max}/T = 2.821\,439\,372...\,c/c_2$	b'	$5.878\,925\,757\ldots \times 10^{10}$	Hz K^{-1}	exact

* The energy of a photon with frequency ν expressed in unit Hz is $E = h\nu$ in J. Unitary time evolution of the state of this photon is given by $\exp(-iEt/\hbar)|\varphi\rangle$, where $|\varphi\rangle$ is the photon state at time $t = 0$ and time is expressed in unit s. The ratio $Et/\hbar$ is a phase.

† The full description of m^{-1} is cycles or periods per meter and that of m is meter per cycle (m/cycle). The scientific community is aware of the implied use of these units. It traces back to the conventions for phase and angle and the use of unit Hz versus cycles/s. No solution has been agreed upon.

‡ Value recommended by the Particle Data Group (Tanabashi, *et al.*, 2018).

§ Based on the ratio of the masses of the W and Z bosons m_W/m_Z recommended by the Particle Data Group (Tanabashi, *et al.*, 2018). The value for $\sin^2\theta_W$ they recommend, which is based on a variant of the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme, is $\sin^2\hat{\theta}_W(M_Z) = 0.231\,22(4)$.

¶ This and other constants involving m_τ are based on $m_\tau c^2$ in MeV recommended by the Particle Data Group (Tanabashi, *et al.*, 2018).

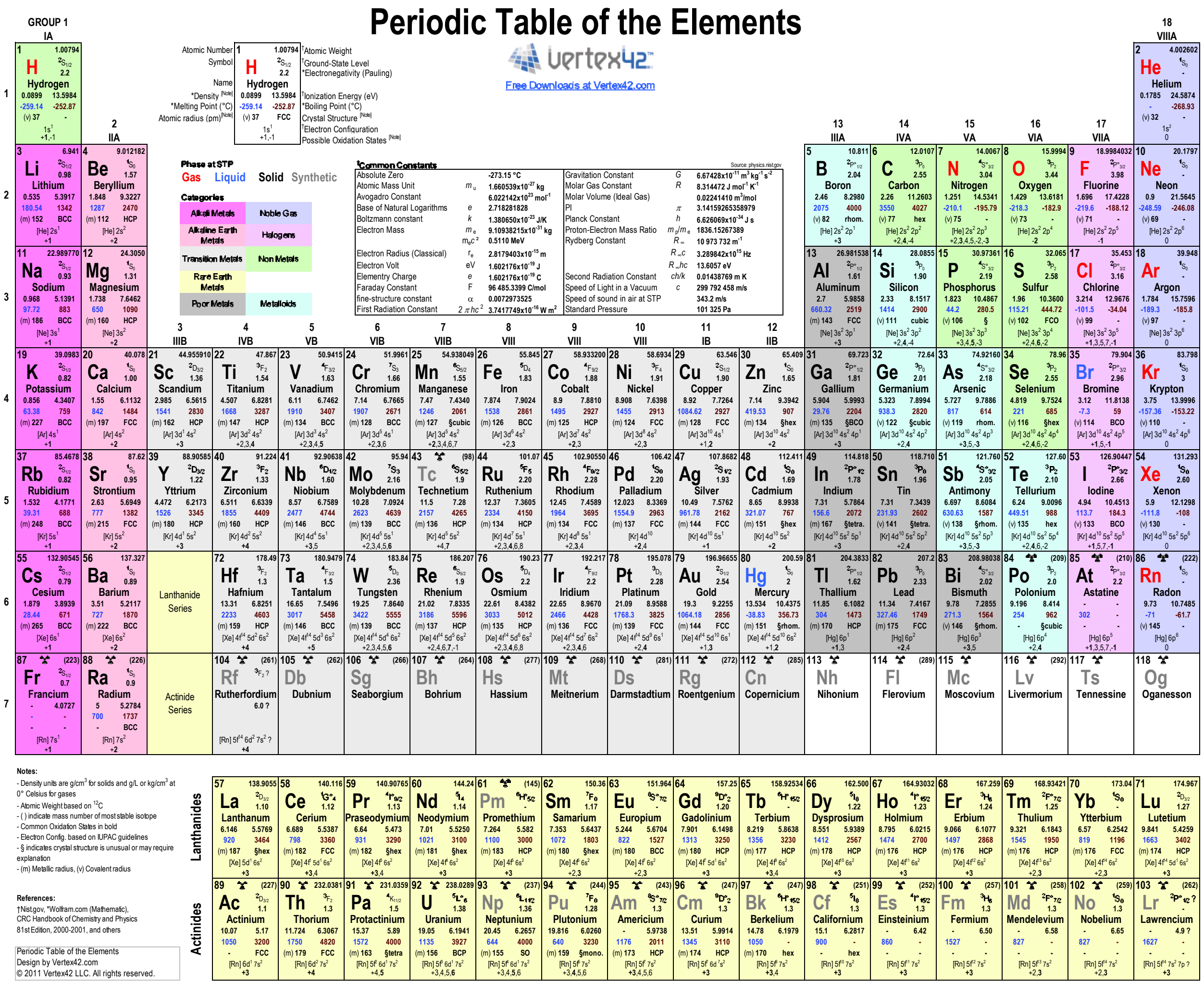
‖ The relative atomic mass $A_r(X)$ of particle X with mass $m(X)$ is defined by $A_r(X) = m(X)/m_u$, where $m_u = m(^{12}\mathrm{C})/12 = 1$ u is the atomic mass constant and u is the unified atomic mass unit. Moreover, the mass of particle X is $m(X) = A_r(X)$ u and the molar mass of X is $M(X) = A_r(X)M_u$, where $M_u = N_A$ u is the molar mass constant and N_A is the Avogadro constant.

** The entropy of an ideal monoatomic gas of relative atomic mass A_r is given by $S = S_0 + \frac{3}{2}R\ln A_r - R\ln(p/p_0) + \frac{5}{2}R\ln(T/\mathrm{K})$.

†† The full description of m^2 is $\mathrm{m}^{-2} \times (\mathrm{m/cycle})^4$. See also footnote for m^{-1}.

Appendix 31

Periodic Table of the Elements


Periodic Table of the Elements
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GROUP 1 IA
18 VIIIA
Phase at STP
Gas Liquid Solid Synthetic
Categories
Alkali Metals
Alkaline Earth Metals
Transition Metals
Rare Earth Metals
Poor Metals
Noble Gas
Halogens
Non Metals
Metalloids
Common Constants
Lanthanide Series
Actinide Series
Lanthanides
Actinides
Period
Notes:
References:
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Further Reading

Mathematics

Analytic Geometry by Murray H. Protter and Charles B. Morrey, Jr.; Addison-Wesley Co., 1966. A standard high school text on analytic geometry.

Ask Marilyn by Marilyn vos Savant. St. Martin's Press, 1992. A selection of Marilyn vos Savant's columns from *Parade* magazine from 1986–1992. Chapter 26 contains the whole discussion of the Monty Hall problem, whose solution she correctly provided in the magazine.

The Circle: A Mathematical Exploration beyond the Line by Alfred S. Posamentier and Robert Geretschlager. Prometheus Books, 2016. An entire book about circles, their properties, and theorems related to circles, written at about the level of these notes.

A College Algebra by Henry Burchard Fine. Ginn and Company, 1904; reprinted by Forgotten Books, 2018. An older, very thorough book on college algebra. This text covers quite a bit of material, and goes into more depth than the algebra covered in these notes.

Conway's Game of Life: Mathematics and Construction by Nathaniel Johnson and Dave Greene. Lulu.com, 2021. A thorough and extensive discussion of Conway's game of Life, including the latest results. A free PDF copy is available at https://conwaylife.com/ .

A Course of Pure Mathematics (3rd ed.) by G.H. Hardy. Dover, 2018. An introductory book on mathematical analysis by one of the great mathematicians of the 20th century. First published in 1908 and reprinted by Dover.

The Drunkard's Walk: How Randomness Rules Our Lives by Leonard Mlodinow. Pantheon Press, 2008. This book, written for the general public, is about the "random walk" problem.

The (Fabulous) Fibonacci Numbers by Alfred S. Posamentier and Ingmar Lehmann. Prometheus Books, 2007. A book that goes into great detail about the Fibonacci numbers, written for the general public.

Fun with Mathematics by Jerome S. Meyer; Fawcett Pub., 1962. A very well-written and enjoyable book on mathematics for the general public.

The Higher Arithmetic: An Introduction to the Theory of Numbers (8th Ed.) by H. Davenport; Cambridge University Press, 2008. A highly regarded and well-respected text on number theory. This is an excellent place to start learning about number theory.

How to Enjoy Calculus by Eli S. Pine; Arco Pub., 1975, 1980. If you've never studied the calculus and are interested learning what it's all about, then this brief book is the place to start. It is the best introductory book on calculus available, bar none. The author states that, "In the years I have taught this subject, no one has gone away without understanding it completely."

Introduction to Geometry (2nd ed.) by H.S.M. Coxeter; Wiley, 1991. A masterpiece of geometrical exposition, by the geometer whom many would consider the greatest of modern times. If you work your way through this text, you will be a serious student of geometry.

An Introduction to the Theory of Numbers (6th ed.) by G.H. Hardy and E.M. Wright; Oxford University Press, 2008. Originally written in 1938, this book continues to be a very well-known and well-respected text on number theory. This book can get into some fairly advanced material, and can be considered one of the definitive texts on number theory. If you're interested in learning more about number theory, this is an excellent place to start.

Mathematics and the Imagination by Edward Kasner and James Newman; Simon and Schuster, 1940. A well-known, classic text on mathematics written for the general public. This is the book that introduced the world to the word *googol*.

Men of Mathematics by E.T. Bell; Simon and Schuster, 1937. A well-known collection of short biographies of famous mathematicians throughout history. Written for a general public audience.

Modern Algebra, Book One and *Modern Algebra, Book Two* by Mary Lociani *et al.*; Houghton-Mifflin Co., 1970 and 1973. — Standard high school texts on basic algebra.

Modern School Mathematics: Geometry by Ray C. Jurgensen, Alfred J. Donnelly, and Mary P. Dolciani; Houghton-Mifflin Co., 1972. A standard high school text on geometry.

The Monty Hall Problem: The Remarkable Story of Math's Most Contentious Brain Teaser by Jason Rosenhouse. A book entirely about the Monty Hall problem, written for the general public.

Numerical Recipes (3rd ed.) by William H. Press, *et al.*; Cambridge University Press, 2007. If you're interested in numerical methods, this is an excellent place to start learning about them. This book does have its detractors, who claim that the methods presented are too simple, and elaborate pre-written black-box routines work better. That's undoubtedly true, but if you want some insights into the basics of numerical methods, this book does that job very well.

Prelude to Mathematics by W.W. Sawyer; Dover Publications, 2011. Another excellent (brief) book on mathematics, intended for a general audience and covering some advanced topics.

The Princeton Companion to Mathematics by Timothy Gowers *et al.*; Princeton University Press, 2008. A massive compilation of all sorts of topics in pure mathematics. Very interesting to browse through and read articles that capture your interest.

The Princeton Companion to Applied Mathematics by Nicholas J. Higham *et al.*; Princeton University Press, 2015. Like its cousin above, but focused on topics in applied mathematics.

The Secrets of Triangles: A Mathematical Journey by Alfred S. Posamentier and Ingmar Lehmann. Prometheus Books, 2012. An entire book dedicated to triangles, their properties, and theorems related to triangles, written at about the level of these notes.

Topology Without Tears by Sidney A. Morris. An excellent starter text on general topology by a well respected mathematician in the field. The book is currently available for free as an e-book on the Internet at `https://topologywithouttears.net/`, along with some accompanying videos. The text is still a work in progress, and is updated from time to time, so be sure you have the latest version. The book is geared toward readers with a wide range of mathematical backgrounds.

The Trachtenberg Speed System of Basic Mathematics by Jakow Trachtenberg; Ishi Press, 2011. A system for performing amazing feats of mental arithmetic, developed by Jakow Trachtenberg while imprisoned in concentration camps during the Holocaust. Be sure to read the amazing foreword to this book — it's an incredible story that could easily be adapted to an action-packed movie.

A Treatise on Algebra (3rd ed.) by Charles Smith; MacMillon and Co., 1892. An older but comprehensive text on college algebra. Beware of books on "algebra": they might be about basic algebra (quadratic equations and so forth), or they might be about so-called "modern algebra," which has to do with abstract concepts like groups, rings, and fields. Smith's book is the former, and covers algebra as we've discussed it here, along with a few related topics. There is absolutely no way to tell from a book's title which kind of "algebra" book it is — you have to look inside.[1] If there's a lot of talk about groups and rings and fields, then it's modern algebra. If there's talk about quadratic equations, then it's the basic algebra we've discussed here.

Trigonometry (2nd ed.) by Elbridge P. Vance; Addison-Wesley, 1969. A standard high school trigonometry textbook.

What is Mathematics? by by Richard Courant and Herbert Robbins; Oxford University Press, 1996. First published in 1941, this is an excellent book on mathematics for a general audience. Albert Einstein said of this book, "A lucid representation of the fundamental concepts and methods of the whole field of mathematics."

The World of Mathematics (4 vol.) by James R. Newman (ed.). Simon and Schuster, 1956. A well-known collection of writings about mathematics, written for a general audience.

Physics

Applied Physics (11th ed.) by D. Ewen, N. Schurter, and P.E. Gundersen; Pearson, 2017. Applied physics is closely related to technical physics. This recent text on applied physics is algebra-based (no calculus) and would make a good companion for this course. The college may consider adopting this text for future offerings of this Technical Physics course.

Brainteaser Physics: Challenging Physics Puzzlers by by Göran Grimvall. Johns Hopkins University Press, 2007. A very interesting and entertaining collection of physics puzzlers, including a discussion of Benford's law.

College Physics by F.W. Sears and M.W. Zemansky; Addison-Wesley, 1960. A good, standard algebra-based textbook on college physics. This text goes into more detail than the present text, and could be covered in roughly two years of college instruction.

Experiments for Technical Physics (2nd ed.) by A. McAlexander; Allyn and Bacon, Inc., 1979. One of very few laboratory manuals written specifically for technical physics. Maybe the *only* such laboratory manual.

The Feynman Lectures on Physics (3 vol.), by R.P. Feynman, R.B. Leighton, and M.L. Sands; Addison-Wesley, 1963. A very well known and respected set of lectures by Nobel laureate Richard Feynman. These

[1] One famous example is the two-volume work *Basic Algebra* by Jacobson. Volume I immediately starts discussing set theory, monoids, groups, rings, ideals, Galois theory, etc. That's modern algebra, and is fairly advanced. On the other hand, *Modern Algebra* by Dolciani is about basic algebra of the kind discussed in this text.

lectures are at an upper undergraduate level, and cover nearly all of physics. These can be found as published paper editions, or free online at `https://www.feynmanlectures.caltech.edu`. There are a number of editions of these lectures available; the author's preference is for the *Commemorative Issue* version, but the most recent is the *New Millennium Edition*. The audio lectures are available on CD-ROM; a problem book is available under the title *Exercises for the Feynman Lectures on Physics*; and a companion book *Feynman's Tips on Physics* is also available.

Technical College Physics, 3rd ed., J.D. Wilson. Saunders Pub., 1992. Now out of print, but available used from online book sellers. This text was once used as the required textbook for this course.

Technical Physics, 3rd ed., F. Bueche. Harper & Row Pub., 1985. Another standard textbook on technical physics. There are not many textbooks on technical physics, and this is one of the few. Out of print, but available used from used bookstores and online used book services.

Technical Physics, P.J. Ouseph, D. Van Nostrand Co., 1980. Another one of just a handful of books on the subject of technical physics.

Thermodynamics, by Enrico Fermi. Dover Publications, 1936, 1956. A brief calculus-based introduction to thermodynamics by one of the giants in the history of physics.

Understanding Thermodynamics by H.C. Van Ness. Dover Publications, 1969, 1983. Another brief calculus-based introduction to thermodynamics.

Remote Sensing

Flattening the Earth: Two Thousand Years of Map Projections by John P. Snyder. University of Chicago Press, 1993. A good, up-to-date history and description of the many map projections in use today.

Introduction to Remote Sensing (6th ed.) by J.P. Campbell R.H. Wynne, and V.A. Thomas; Guilford Press, 2023. A comprehensive and up-to-date text on remote sensing.

Remote Sensing Laboratory Manual by Floyd F. Sabins; Kendall/Hunt Publishing Co., 1997. A good manual of laboratory exercises in remote sensing.

Reference Works

CRC Standard Mathematical Tables and Formulas (33rd ed.) by Dan Zwillinger (ed.); CRC Press, 2018. A very well-known reference work on many areas of mathematics. There's quite a bit of interesting and useful reference material in here.

CRC Handbook of Chemistry and Physics (104th ed.) By John R. Rumble (ed.), CRC Press, 2023. A large and very well-known standard reference, with lots of information related to physics and chemistry. Published in a new edition every year.

Handbook of Mathematical Functions by Milton Abromowitz and Irene A. Stegun; Dover Publications, 1965. This highly regarded reference work, popularly known as "Abromowitz and Stegun," is still in print after its initial publication by the National Bureau of Standards in 1964. Lots of tables and other information on special functions, integrals, etc.

Table of Integrals, Series, and Products (8th ed.) by I.S. Gradshteyn and I.M. Ryzhik; D. Zwillinger (ed.). Especially for students of calculus, you'll need a good table of integrals. This one is very well known and quite comprehensive, and includes tables of series and products as well.

On-Line Resources

Online Encyclopedia of Integer Sequences (OEIS)
https://oeis.org/

Wolfram|Alpha
https://www.wolframalpha.com/

Wolfram MathWorld
https://mathworld.wolfram.com/

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[1] John Chadwick, *The Decipherment of Linear B*. (Cambridge, 1958).

[2] Margalit Fox, *The Riddle of the Labyrinth: The Quest to Crack an Ancient Code*. (Ecco Pub., 2013.)

[3] E. Gelin, *Éléments de Trigonométrie plane et sphérique à l'usage des élèves des Cours professionnels des candidats aux Écoles spéciales des Universités et à l'École militaire de Bruxelles* (1888).

[4] John Leslie, *The Philosophy of Arithmetic; Exhibiting a Progressive View of the Theory and Practice of Calculation, with Tables for the Multiplication of Numbers as Far as One Thousand*. Abernethy & Walker, Edinburgh, 1820.

[5] Jean Meeus, *Astronomical Algorithms* (2nd ed.) (Willmann-Bell, 1998).

[6] Tom Siegfried, In honor of his centennial, the Top 10 Feynman quotations, *Science News*, May 11, 2018.

[7] Marinos Yeroulanos, *A Dictionary of Classical Greek Quotations*. I.B. Tauris (2016).

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