Physics Recreations:
The Slide Rule

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1 Introduction

Before electronic calculators became widely available around 1975, students, engineers, and scientists performed mathematical calculations using an instrument called a *slide rule*. With this simple device, you could to multiply, divide, calculate reciprocals, squares, square roots, cubes, cube roots, logarithms, sines, cosines, tangents, cotangents, inverse trigonometric functions, and compute powers of numbers. Here we'll build a simple slide rule and show how to use it.

2 Obtain a Slide Rule

You'll need a slide rule to practice with. We'll use a paper slide rule kit from the May 2006 issue of *Scientific American*, which is duplicated on the last page of this handout. It is very similar to the type used decades ago, and provides good practice in using analog devices.

Also, an excellent software slide rule simulator is available on the Internet at `http://homepages.slingshot.co.nz/~timb3000/index.html`. You can obtain a real slide rule on the Internet from on-line auction sites or from Sphere Research Corporation (`http://sphere.bc.ca/test/sruniverse.html`). Some popular top-of-the-line models were the Post Versalog 1460, the K+E Deci-Lon, and the Faber-Castell 2/83N.

3 Instructions

The slide rule has three major limitations compared to electronic calculators:

- It cannot add or subtract. Addition and subtraction must be done using paper and pencil.
- It does not keep track of the decimal point; you locate the decimal point yourself by estimating the answer in your head.

- It can perform calculations to only three or four significant digits.

The slide rule consists of three parts: (1) the *body* (or *stock*); (2) the *slide* (which moves left and right within the body); and (3) the *cursor* (the transparent sliding window with a hairline), which is used to help read the scales. Inscribed on the body and slide are sets of scales. The slide rule from *Scientific American* has nine scales; some more advanced models have 20, 30, or even more. Scales are generally labeled with one or two letters, and these names are standard on most rules.

On the *Scientific American* slide rule, the T, K and A scales are on the upper part of the body; the B, CI, and C scales are on the slide; and the D, L, and S scales are on the lower part of the body.

1. **The C and D Scales.** The C and D scales are the scales that are used most often: they are used to perform multiplication and division.

   Multiplication. Set the left *index* (the left 1) on the C scale over the first number (the multiplicand) on the D scale. Find the second number (the multiplier) on the C scale; move the cursor so that the hairline is over this second number. You then read the product on the D scale under the hairline. If the second number on C is beyond the right-hand end of D, then move the slide to the left and use the right index (the right 1) instead of the left index of C. (Try $2 \times 3 = 6$. Note that this same setting also represents 20×3, 20000×0.03, 0.2×30, etc. You place the decimal place in the result by estimating the answer in your head.)

   Division. Division is actually a bit easier than multiplication. Set the hairline in the cursor over the dividend (the numerator) on the D scale. Then move the slide so that the divisor (the denominator) on the C scale is also under the hairline. The quotient will then be found on the D scale, under either the left or right index of the C scale. (Try $6 \div 2 = 3$.)

2. **The CI Scale.** The CI scale is a reversed C scale; it shows the reciprocals of numbers on the C scale. To find the reciprocal of a number, just set the hairline over the number on the C scale, and read its reciprocal on the CI scale. (Notice that numbers on the CI scale run backwards, increasing from right to left.) (Try $1/4 = 0.25$.)

   One trick of skilled users is to use the CI and D scales for multiplication, instead of the C and D scales, so that you compute $x \times y$ as $x \div (1/y)$. To do this, set the hairline in the cursor over the multiplicand on the D scale. Then move the slide so that the multiplier on the CI scale is also under the hairline. The product will then be found on the D scale, under either the left or right index of the C scale. This trick makes it possible to do multiplication without worrying about running over the right end of the D scale.

3. **The A and B Scales.** These scales are used to find squares and square roots.

   Squares. To square a number, place the hairline over the number on the D scale, and find its square on the A scale. (You could also place the hairline over the number on the C scale, and find its square on the B scale.) (Try $4^2 = 16$.)

   Square roots. To find the square root of a number, place the hairline over the number on the A scale, and read its square root on the D scale. Since the A scale has two scales on it, you have to know which half of the scale to use. You can use this rule: write the number in scientific notation. If the exponent of 10 is even, use the left half of A; if it is odd, use the right half. (Try $\sqrt{9} = 3$, and $\sqrt{60} = 7.75$.)

You can also multiply and divide using the A and B scales, but with reduced accuracy (due to the smaller scales).

4. **The K Scale.** The K scale is used to find cubes and cube roots.

   Cubes. To find the cube of a number, place the hairline over the number on the D scale, and read its cube on the K scale. (Try $2^3 = 8$.)

   Cube roots. To find the cube root of a number, place the hairline over the number on the K scale, and read its cube root on the D scale. Since the K scale is divided into three parts, you will have to take care to use the correct third of the K scale when doing this. Write the number in scientific notation; if the exponent of 10 is a multiple of 3, then use the left third; if it is 1 more than a multiple of 3, use the middle third; if it is 2 more than a multiple of 3, then use the right third. (Try $\sqrt[3]{27} = 3$.)

5. **The L Scale.** The L scale is used to calculate common (base 10) logarithms. The L scale will only show the part of the logarithm to the right of the decimal point. You must provide the part to the left of the decimal point from knowing the magnitude of the number.

   Place the hairline over a number on the D scale, and read the logarithm (the part to the right of the decimal point) on the L scale. (Try $\log 300 = 2.477$. Since $300 = 3 \times 10^2$, the exponent of 10 (2) gives the part to the left of the decimal. The part to the right of the decimal (0.477) is read on the L scale.)

   To find natural logarithms, use $\ln x = \log x / \log e = 2.30 \log x$. In other words, multiply the base 10 logarithm by 2.30.

6. **The S Scale.** The S scale is used to find sines and cosines of angles.

   Sine of an angle between 0° and 5.°74. the sine of an angle in this range is approximately the angle converted to radians, so just multiply the degrees by $\pi/180 = 0.0175$.

   Sine of an angle between 5.°74 and 90°. Set the hairline over the angle (in degrees) on the S scale (using the black numbers), and read its sine under the hairline on the D scale. (Try $\sin 30° = 0.5$.)

   Cosine of an angle between 0° and 84.°3. Set the hairline over the angle (in degrees) on the S scale (using the grey numbers), and read its cosine under the hairline on the D scale. (Try $\cos 30° = 0.866$.)

   Cosine of an angle between 84.°3 and $90°$. Use $\cos\theta \approx (90° - \theta) \times (\pi/180)$.

7. **The T Scale.** The T scale is used to find tangents and cotangents of angles.

   Tangent of an angle between 0° and 5.°74. The tangent of an angle in this range is approximately the angle converted to radians, so just multiply the degrees by $\pi/180 = 0.0175$.

   Tangent of an angle between 5.°74 and 45°. Set the hairline over the angle (in degrees) on the T scale (black numbers), and find its tangent under the hairline on the D scale. (Try $\tan 30° = 0.577$.)

Tangent of an angle between 45° and 84.°3. First, align the C and D scales (so that the slide is centered). Set the hairline over the angle (in degrees) on the T scale (grey numbers), and find its tangent under the hairline on the CI scale. (Try $\tan 60^\circ = 1.73$.)

Cotangent of an angle between 5.°74 and 45°. First, align the C and D scales (so that the slide is centered). Set the hairline over the angle (in degrees) on the T scale (black numbers), and find its cotangent under the hairline on the CI scale. (Try $\cot 30^\circ = 1.73$.)

Cotangent of an angle between 45° and 84.°3. Set the hairline over the angle (in degrees) on the T scale (grey numbers), and find its cotangent under the hairline on the D scale. (Try $\cot 60^\circ = 0.577$.)

Cotangent of an angle between 84.°3 and 90°. Use $\cot\theta \approx (90^\circ - \theta) \times (\pi/180)$.

Combining Operations

Skilled slide rule operators can often set the slide rule to perform several operations at once. For example:

- $x \times y \div z$. Do the division first: set the hairline over y on the D scale, then move the slide so that z on the C scale is also under the hairline. Now move the hairline over x on the C scale and read the result on the D scale. (Try $8 \times 3 \div 4 = 6$.)
- $x \times y \times z$. Set the hairline over x on the D scale, then move the slide to place y on the CI scale under the hairline. Move the hairline to z on the C scale, and read the product under the hairline on the D scale. (Try $1.2 \times 2.3 \times 6.4 = 17.66$.)
- $x \times y^2$. Move the slide to put the index (1) on the C scale over the number that is squared (y) on D scale. Move the hairline over the number that is *not* squared (x) on the B scale, and read the result on the A scale. (Try $2 \times 3^2 = 18$.)
- x^3 *and* $x^{3/2}$. If a K scale is not available, the previous method may be used to compute cubes using only the A, B, C, and D scales. Move the slide to put the index (1) on the C scale over x on D scale. Move the hairline over x on the B scale, and read the result on the A scale. (Try $2^3 = 8$.) This method also gives $x^{3/2}$: just read $x^{3/2}$ under the hairline on the D scale. (Try $2^{3/2} = 2.83$.)

Numbers to Powers

Suppose you wish to take a number to an arbitrary power (i.e. y^x). Sophisticated slide rules have a set of "log-log" scales for computing this, but it can also be done on the *Scientific American* rule, using the relation

$$y^x = 10^{x \log y}.$$

Suppose, for example, we wish to find $2.3^{4.6}$. Using the L scale, we find $\log 2.3 = 0.362$; then using the C and D scales, we find $x \log y = 4.6 \times 0.362 = 1.664$. Now we need to compute the antilog, $10^{1.664}$ by looking up 0.664 on the L scale, and reading 46.1 on the D scale. Hence $2.3^{4.6} = 46.1$.

As a common special case,

$$e^x = 10^{0.434x}.$$

Gauge Points

Many slide rules have special marks called *gauge points* on some of the scales. A few common gauge points found on the C and D scales are:

- π. This is just $\pi = 3.14$, and is used to multiply and divide by π.
- *C*. This is at $\sqrt{4/\pi} = 1.128$, and is used to compute areas of circles. To compute the area of a circle, set the gauge mark C on the C scale over the circle diameter d on the D scale, and read the area A on the A scale under the index of B.
- R or ρ°. This is at $180/\pi = 57.3$, and is used to convert between degrees and radians. (Radians $\times\ \rho^\circ =$ degrees.)

Advanced Techniques

A number of advanced slide rule techniques have been documented (see, for example, the book *The Slide Rule* by C.N. Pickworth). A couple of examples are shown here.

1. *Cube roots.* The slide rule may be used as a kind of analog computer to compute cube roots using only the A, B, C, and D scales. Place the hairline over the number whose cube root is to be extracted on the A scale. Now move the slide back and forth until the number on the D scale under the index on the C scale matches the number on the B scale under the hairline. This is a trial-and-error method, where you will move the slide back and forth until you read the same numbers on both B and D.

   To use this method, you need to know which half of the A scale to set the original number on, and which index to use. The rule is: write the number in scientific notation. If the power of 10 is a multiple of 3, then use the left-hand side of A and the left index of C. If it is 1 more than a multiple of 3, then use the right-hand side of A and the left index of C. If it is 2 more than a multiple of 3, then use the left-hand side of A and the right index of C.

2. *Solving equations.* Occasionally the slide rule can be used as an analog computer to solve an equation. For example, to find the value of x that satisfies $e^x = 1/x$, we first note that the root lies between $x = 0.5$ and $x = 1$ (since $e^{0.5} < 1/0.5$, but $e^1 > 1/1$), and so the number on both sides of the equation must be between 1 and 2. Align the slide of the slide rule with the body (so the C and D scales line up), then move the cursor back and forth until the same number appears under the hairline on both the CI and LL2 scales (see below for a description of the LL scales). We find solution when 1.763 appears under the hairline on both scales, so the solution is then read on the D scale: $x = 0.567$.

Other Scales

Here are some other scales you may encounter on other slide rules (but are not on the *Scientific American* slide rule):

1. **The CF and DF Scales.** These are“folded” C and D scales, with the index (1) in the middle. This can make it more convenient to do multiplication: if the

multiplicand runs off the right end of the slide rule, you can find it on the CF scale instead, and read the product on the DF scale.

To put the index *exactly* in the middle of the scale, it should be located at $\sqrt{10} = 3.16$ on the C and D scales. But since $\sqrt{10} \approx \pi$, the scales are folded at π instead, so that the CF and DF scales can do double duty: they can also be used to multiply and divide by π. (Numbers on DF are π times numbers on D, and similarly with CF and C.)

2. The **DI, CIF, DIF, and BI Scales.** These are additional inverted scales, similar to the CI scale. Just as the CI scale shows reciprocals of the numbers on the C scale, these scales give reciprocals of numbers on the D, CF, DF, and B scales, respectively. The CIF scale is often present if CF and DF are on the slide rule; the DIF and BI scales are uncommon. Note that all of these scales are read from right to left.

3. **The ST (or SRT) Scale.** These scales give either sines or tangents of angles between $0^\circ\!.573$ and $5^\circ\!.74$. In this range of angles, the sine and tangent are nearly equal (to slide rule accuracy). If this scale is not present, you can get an approximate answer by multiplying the angle by 0.0175, as described in the descriptions of the S and T scales. (This scale can also be used to convert angles in this range from degrees to radians.)

4. **The R$_1$ and R$_2$ (or W$_1$ and W$_2$) Scales.** These are square root scales—actually one long scale, twice the length of the C and D scales, but divided into two parts. You can use them to find square roots more accurately than with the A and B scales. Sometimes the A and B scales are omitted if the R$_1$ and R$_2$ scales are present.

5. **The LL (Log-Log) Scales.** If these scales are present, there will be several of them, with names like LL0, etc. These are used to find e^x (e to the power of the number on the D scale), and also to take numbers to arbitrary powers. Some of the scales will be for e to positive powers, and other scales are for e to negative powers. The whole set of scales typically covers e^{-10} to e^{+10} (0.000045 to 22,000).

   To find y^x, put the hairline over y on the appropriate LL scale, move the index on the C scale under the hairline, then move the hairline over x on the C scale, and read y^x on one of the LL scales.

6. **The P Scale.** This is a "Pythagorean" scale that gives $\sqrt{1 - D^2}$, where D is a number on the D scale. If the sine of a number is on the D scale, then its cosine is on the P scale (and vice versa).

7. **The Sh, Ch, and Th Scales.** These are for computing hyperbolic functions (hyperbolic sine, cosine, and tangent, respectively).

(References: "When Slide Rules Ruled" by Cliff Stoll, *Scientific American*, May 2006; and *The Slide Rule* by C.N. Pickworth.)

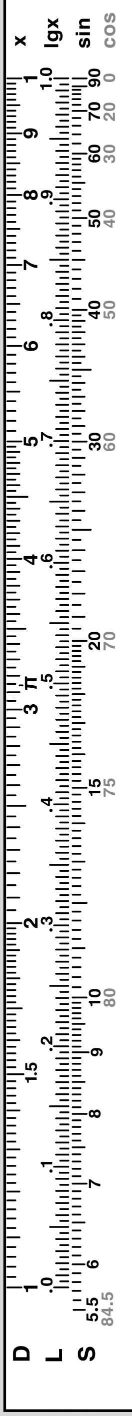
4 Exercises

Use the slide rule to calculate the following:

15×17	=	______________
27×45	=	______________
$6 \div 4.5$	=	______________
4.3^2	=	______________
$\sqrt{45}$	=	______________
2.3^3	=	______________
$\log_{10} 37.0$	=	______________
$\sin 22°$	=	______________
$\cos 52°$	=	______________
$\tan 23°$	=	______________
$1.48^{3.88}$	=	______________

a Cut here

c Cut here



CHRIS HAMANN AND NANCY SHAW

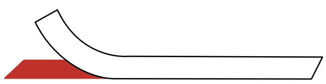
ASSEMBLY INSTRUCTIONS

1 Cut out the entire white panel (*a*). Cut along line between parts A and B (*b*), then remove excess (*c*).

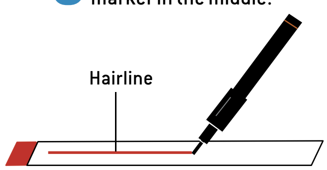
2 Fold part A along the dotted lines.

3 Slip part B into the folded part A.

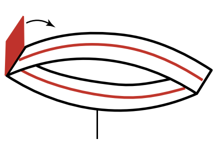
4 To make the cursor (the sliding window that is inscribed with a hairline), use the guides to the left to measure two pieces of transparent tape. Make one section the length of the black line and the other the length of the red line. Place the adhesive sides together.



5 Draw a line with a fine marker in the middle.

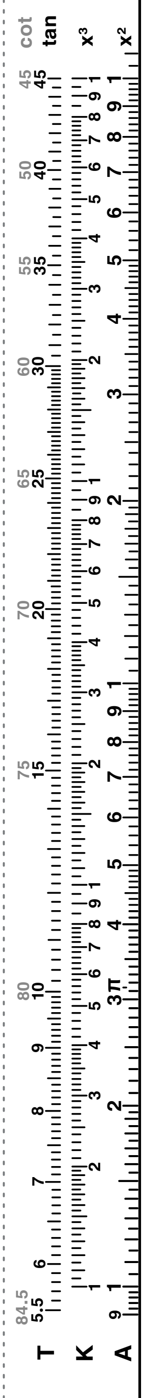


6 Wrap the folded tape around the slide rule for sizing. Use the adhesive end to complete the cursor. Slide cursor onto the rule.

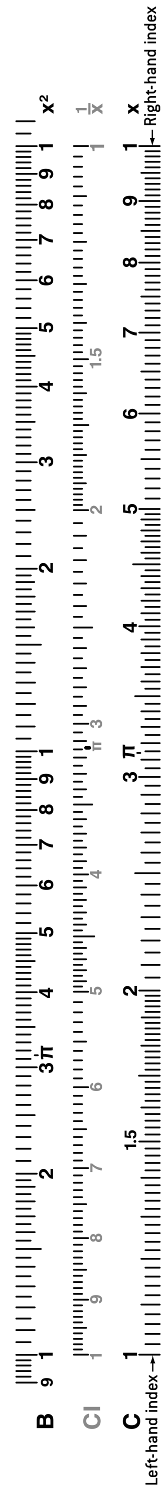


Completed cursor

Part **A**



b Cut here



(Slider)

Part **B**