Physics Recreations:
The Doomsday Algorithm

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Introduction

The *doomsday algorithm* is a clever method for determining the day of week for any calendar date, which you can do entirely in your head. The method is surprisingly easy to learn and remember, and will allow you to calculate the day of the week for any date on the Gregorian calendar in just a few seconds.

The calendar we use today is called the *Gregorian calendar*, and was adopted in the United States (when we were still British colonies) on Thursday, September 14, 1752; prior to that date we were on the old *Julian calendar*. The method we'll look at first only works for dates on the current Gregorian calendar; we'll look later at the modification needed to calculate dates on the Julian calendar.

The doomsday algorithm was developed by noted mathematician John Horton Conway, perhaps best known for his cellular automata game "Life." To begin, Conway defines what he, for some reason, calls "doomsday":

"Doomsday" is the day of week of the last day of February.

That is, "doomsday" is the day of week of February 28 (or February 29 in a leap year). The general idea of the algorithm is to determine doomsday for the year of interest, then to identify a doomsday in the month of interest. Moving forward or backward in jumps of 7 days gives other doomsdays in that month; when near the date of interest, just count out individual days to find the day of week.

Even Months

Let's begin with even-numbered months (2, 4, 6, 8, 10, 12, for February, April, June, August, October, and December). It turns out, by an interesting coincidence, that in even-numbered months (except February), the date that is the same number as the month is always a doomsday. So, for example, the last day of February in 2011 (Feb. 28) falls on a Monday, so doomsday for 2011 is Monday. For the other even-numbered months, 4/4, 6/6, 8/8, 10/10, and 12/12 are also doomsdays, so they also fall on Monday. (That is, April 4, June 6, August 8, October 10, and December 12.) Knowing that, you can figure out the day of the week of any date by adding or subtracting multiples of 7 days to get near the date, then count out individual days.

Example. On what day of the week is Halloween in 2011? *Solution.* Doomsday for 2011 is Monday; therefore 10/10 (October 10) is also a doomsday (Monday); if we add $7 \times 3 = 21$ days to the 10th, we get October 31, which must also be a Monday.

Example. On what day of the week is April 15, 2011? *Solution.* Again we already know that doomsday for 2011 is Monday. Then 4/4 (April 4) is also a doomsday (Monday). Adding 7, we therefore know that April 11 must be on a Monday, and therefore April 15 (four days later) must be on a Friday.

Example. On what day of the week is Christmas Day in 2011? *Solution.* We know that doomsday for 2011 is Monday. Therefore 12/12 (December 12) is also a doomsday (Monday). If we add 7 days, we find December 19 is also on a Monday, and Christmas Day is 6 days later, so it must be on a Sunday.

Odd Months

So now we know how to do even-numbered months. What about odd-numbered months? These would be 1, 3, 5, 7, 9, and 11 (i.e. January, March, May, July, September, and November). Conway came up with a clever mnemonic device for odd-numbered months 5 through 11:

"I work 9 to 5 at the 7-Eleven."

This is supposed to remind you that for odd-numbered months (forgetting January and March for the moment), 9/5, 5/9, 7/11, and 11/7 are all doomsdays. (That is, September 5, May 9, July 11, and November 7.)

We've left out March. We know doomsday is the day of the week of the last day of February, which we can think of as March 0, so of course March 7 will also be a doomsday.

We've also left out January. For January, doomsday is January 3, or January 4 during leap year. This is easy to remember: it's January 3 during the 3 common years of a four-year cycle, or January 4 during the fourth (leap) year.

Example. On what day of the week is July 4, 2011? *Solution.* We already know that doomsday for 2011 is Monday. Therefore 7/11 (July 11) is also a doomsday (Monday). Then 7 days earlier (July 4) is also on a Monday.

Example. On what day of the week is May 5, 2011? *Solution.* Since doomsday for 2011 is Monday, we know that 5/9 (May 9) is also a doomsday (Monday). Subtracting 7 days, this means May 2 is a Monday, and so May 5 is a Thursday.

Example. On what day of the week is New Year's Day in 2011? *Solution.* Since doomsday for 2011 is Monday, we know that January 3 is also a Monday, and so New Year's Day (two days earlier) is a Saturday.

Other Years

All of the examples so far have been for 2011. What about other years? First, let's consider the 20th century. The key thing to remember for the 20th century is:

Doomsday for 1900 is Wednesday. (If you were born in the 1900s, you can remember this as "$1900 =$ We-in-dis-day".)

Then for any year of the form $19yy$, find:

1. The number of 12s in yy;
2. The remainder from step 1; and
3. The number of 4s in the remainder in step 1.

Add these three numbers together and subtract multiples of 7 to get a number between 0 and 6. Then add that number of days to doomsday for 1900 (Wednesday) to find doomsday for that year, then continue as before.

Example. The armistice ending World War I was signed on November 11, 1918. What day of the week was this? *Solution.* Going through the calculation just described, $yy = 18$; there is one 12 in 18, with a remainder of 6, and there is one 4 in the remainder. Adding these, we get $1 + 6 + 1 = 8$; subtracting 7, we get 1. Since doomsday for 1900 is Wednesday, we know doomsday for 1918 is one day after Wednesday, or Thursday. Now we know 11/7 is also a doomsday (Thursday), and so November 11 is four days later, on a Monday.

Example. On what day of the week was Pearl Harbor attacked? (You may well already know the answer if you know American history.) *Solution.* The attack on Pearl Harbor, Hawai‘i was on December 7, 1941. In this case $yy = 41$. There are 3 12s in 41, with a remainder of 5, and there is a single 4 in this remainder. The sum of the three numbers is $3 + 5 + 1 = 9$, and we can subtract a single 7 to get 2. Recall that doomsday 1900 is Wednesday, so doomsday for 1941 is two days later (Friday). Therefore 12/12 (December 12) is also a doomsday (Friday), and so a week earlier (December 5) was also a Friday, and so two days later, December 7, was a Sunday.

For the 21st century (years of the form $20yy$), note that **doomsday for 2000 is Tuesday** (“2000=Tue.”), then proceed as before.

Example. On what day of the week is May 15, 2075? *Solution.* Here the last two digits $yy = 75$, which contains 6 12s, with a remainder of 3; there are no 4s in this remainder. Therefore $6 + 3 + 0 = 9$; subtract 7 and we have 2, so doomsday for 2075 is two days after Tuesday, or Thursday. So 5/9 (May 9) is also on a Thursday, and May 15 is 6 days later, or Wednesday.

Other Centuries

For other centuries, simply note the doomsdays for these century years, and proceed as with years of the form $19yy$ or $20yy$.

- 1700: Sunday
- 1800: Friday
- 1900: Wednesday
- 2000: Tuesday

The cycle repeats after this: 2100 is Sunday; 2200 is Friday, etc. The Gregorian calendar repeats after 400 years, so you can always add or subtract multiples of 400 years to bring the date into the range 1700–2099 to find the day of week.

Example. On what day of the week was November 17, 1858? *Solution.* The last two digits of the year $yy = 58$, which contains 4 12s, with a remainder of 10; there are 2 4s in this remainder. Therefore $4 + 10 + 2 = 16$; subtract two 7s and we have 2, so doomsday for 1858 is two days after doomsday for 1800 (Friday), which is Sunday. Then we know 11/7 is also a doomsday (Sunday); adding 7 days, we know 11/14 must therefore also be a Sunday, and so the 17th (3 days later) must be a Wednesday.

The Julian Calendar

Everything discussed up until now is for the Gregorian calendar, which is valid for dates of Thursday, September 14, 1752 and later (in the U.S. and the U.K.). The day before this—the last day we were on the Julian calendar—was called Wednesday, September 2, 1752. In switching from the Julian to the Gregorian calendar, we changed the leap-year rule and also dropped 11 days from the calendar, so there was no September 3–13 in 1752.

For dates on the Julian calendar (on or before September 2, 1752), we need to modify the century rule for determining doomsday. For the Julian calendar:

- 1000: Thursday
- 1100: Wednesday
- 1200: Tuesday
- 1300: Monday
- 1400: Sunday
- 1500: Saturday
- 1600: Friday

The cycle repeats after this, in both directions: 1700 is Thursday, 900 is Friday, 800 is Saturday, etc. Notice the pattern: for every century you go *back* in time, doomsday for that century year moves *ahead* by one day. Since the Julian calendar repeats every 700 years, you can always add multiples of 700 years to the date to bring it into the range 1000–1699, then use the table above.

Conway suggests a way to remember a starting point: "1000 = THOUSday (Thursday)."

Example. On what day of the week was Battle of Hastings (October 14, 1066)? *Solution.* This date is on the Julian calendar, so we use the century rules for Julian dates: doomsday for 1000 is Thursday. The last two digits of the year $yy = 66$, which contains 5 12s, with a remainder of 6; there is 1 4 in this remainder. Therefore $5 + 6 + 1 = 12$; subtract 7 and we have 5, so doomsday for 1066 must be 5 days after doomsday for 1000: Thursday plus 5 days is Tuesday. We know 10/10 must therefore also be a doomsday (Tuesday), and so 4 days later (October 14) must have been on a Saturday.

B.C. Dates

All B.C. dates are on the old Julian calendar. There was no year 0; the year before A.D. 1 was 1 B.C. To treat B.C. dates properly with these algorithms, you will need to convert the year into an equivalent non-positive number. For example, change 1 B.C. to 0, 2 B.C. to the year -1, etc. In general, subtract 1 from the B.C. year and put a minus sign on it; then add multiples of 700 years to bring the year into the range 1000–1699, and use the table shown in the previous section.

Example. On what day of the week was Julius Caesar assassinated (March 15, 44 B.C.)? *Solution.* This is a B.C. date, so we use the Julian calendar. The year 44 B.C. is equivalent to -43; let's add 700 years to get A.D. 657. This isn't enough to get to the range 1000–1699, so let's add 700 again to get A.D. 1357. So the calendar for 44 B.C. is the same as the calendar for A.D. 1357. Now as seen in the previous section, doomsday for 1300 is Monday. The last two digits of the year $yy = 57$, which contains 4 12s, with a remainder of 9; there are 2 4s in the remainder. Therefore $4 + 9 + 2 = 15$; subtract two 7s to get 1, so doomsday for 1357 is one day after doomsday for 1300: Monday plus 1 day is Tuesday. We know that March 7 must therefore also be a doomsday (Tuesday), and so 1 week later (March 14) is also a Tuesday. The next day, March 15, was therefore a Wednesday.

Summary

To find the day of week for a given date (here “doomsday” means the day of week of the last day of February):

1. Determine doomsday for the beginning of the century of interest: 1900=Wednesday, 2000=Tuesday. (See text for other centuries.)
2. Determine doomsday for the year of interest. To do this:

   Add together: (a) the number of 12s in the last two digits of the year, (b) the remainder; and (c) the number of 4s in the remainder. Subtract 7 until you have a number between 0 and 6, then add this many days to the century doomsday in step 1. The result is that year’s doomsday.
3. Find a doomsday in the month of interest (leap year values in parentheses):
   (a) *Even months:* 2/28(29), 4/4, 6/6, 8/8, 10/10, 12/12
   (b) *Odd months:* 1/3(4), 3/7, 5/9, 7/11, 9/5, 11/7 (“I work 9 to 5 at the 7-Eleven.”)
4. Now add or subtract multiples of 7 days to get other doomsdays in that month until near the date of interest, then count out individual days to find the day of week of the desired date.

Exercises

For fun, you might like to see if you can find, in your head, the day of the week for each of these dates:

1. The day you were born.
2. The signing of the Mayflower Compact on November 11, 1620 (Julian calendar).
3. The adoption of the Declaration of Independence on July 4, 1776.
4. The attack on Fort Sumter on April 12, 1861.
5. The assassination of President Lincoln on April 14, 1865.
6. The first successful test of Edison's electric light on October 22, 1879.
7. The shooting of President Garfield on July 2, 1881.
8. The first Wright Brothers flight on December 17, 1903.
9. The sinking of the *Titanic* on April 15, 1912.
10. The stock market crash on October 29, 1929.
11. The crash of the airship *Hindenburg* on May 6, 1937.
12. D-Day: June 6, 1944.
13. The date Marty McFly arrives in the past in the movie *Back to the Future*: November 5, 1955.
14. The launch of the Soviet Sputnik satellite on October 4, 1957.
15. The assassination of President Kennedy on November 22, 1963.
16. The first Moon landing on July 20, 1969.
17. The resignation of President Nixon on August 9, 1974.
18. America's Bicentennial (July 4, 1976).
19. The first test of the time machine in the movie *Back to the Future*: October 25, 1985.
20. The first day of the 21st century (January 1, 2001).
21. September 11, 2001.
22. The "end of the Mayan calendar" on December 20, 2012.
23. Marty McFly's arrival in the future in the movie *Back to the Future II*: October 21, 2015.
24. "Doomsday," according to a 1960 paper in the journal *Science*:[1] November 13, 2026.
25. The date of "first contact" in the movie *Star Trek: First Contact*: April 5, 2063.

[1] The authors did a curve fit to historical population data; the resulting equation predicted that the Earth's population would become infinite on this date. (von Foerster, Mora, and Amiot, *Science*, **132** (3436), 1291-1295 (1960).)

References

- "What Day Is Doomsday? How to Mentally Calculate the Day of the Week for Any Date" by Chamberlain Fong. *Scientific American* Web site, October 18, 2011: `http://www.scientificamerican.com/article.cfm?id=calendar-algorithm`
- `http://rudy.ca/doomsday.html`
- "The Doomsday Rule" in Chapter 24 of *Winning Ways for Your Mathematical Plays* (Vol. 4) by E.R. Berlekamp, J.H. Conway, and R.K. Guy (A.K. Peters, 2004).

Solutions to Exercises

(2) Sat.; (3) Thur.; (4) Fri.; (5) Fri.; (6) Wed.; (7) Sat.; (8) Thur.; (9) Mon.; (10) Tue.; (11) Thur.; (12) Tue.; (13) Sat.; (14) Fri.; (15) Fri; (16) Sun.; (17) Fri.; (18) Sun.; (19) Fri.; (20) Mon.; (21) Tue.; (22) Thur.; (23) Wed.; (24) Fri.; (25) Thur.