The Coriolis Force

D.G. Simpson, Ph.D.
Department of Physical Sciences and Engineering
Prince George's Community College

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Introduction

Imagine you're on a rotating merry-go-round, and you throw a ball to another person who's on the opposite side of the merry-go-round. If you aim directly at the other person, you'll miss them—the ball will travel in a straight line relative to the ground, but the merry-go-round will have rotated during the time the ball is in the air. Relative to the merry-go-round, the ball will appear to move along a curved path. You can attribute this curvature to a "fictitious force" called the *Coriolis force*. The Coriolis force is not a real force—it's just an artifact of viewing the ball's motion in a rotating reference frame. The ball really moves in a straight line relative to the ground.

But suppose you're in the rotating reference frame of the merry-go-round. You'll see the ball move in a curved path, which can't happen unless there is a "force" present. We can compute the magnitude of this Coriolis force by considering the following situation. Suppose you're at the center of the merry-go-round, and throw a ball outward with velocity v while the merry-go-round is rotating with an angular velocity Ω. After a time t, the ball will have moved a radial distance $r = vt$. At time t, a point on the merry-go-round a distance r from the center will have moved a distance of arc length

$$
\begin{aligned}
s &= r\theta && (1)\\
&= r(\Omega t) && (2)\\
&= (vt)\Omega t && (3)\\
&= \Omega v t^2. && (4)
\end{aligned}
$$

But under a constant acceleration a_c, we know

$$s = \frac{1}{2} a_c t^2. \tag{5}$$

Comparing Eq. (4) with Eq. (5), we deduce that the Coriolis acceleration a_c is given by

$$a_c = 2\Omega v. \tag{6}$$

More generally, in terms of vectors, the Coriolis acceleration vector $\mathbf{a}_c$ is given by

$$\boxed{\mathbf{a}_c = -2\,(\boldsymbol{\Omega}\times\mathbf{v})} \tag{7}$$

From Newton's Second Law, the corresponding Coriolis force $\mathbf{F}_c$ on a body of mass m is then

$$\boxed{\mathbf{F}_c = -2m\,(\boldsymbol{\Omega}\times\mathbf{v})} \tag{8}$$

Examples

Golf

For example, suppose we're on the surface of the Earth, in the northern hemisphere, and hit a golf ball due south with velocity v. Since the Earth rotates to the east, the Earth's angular velocity vector $\mathbf{\Omega}$ is along the Earth's axis, northward out of the north pole. Then by Eq. (8), there will be a *westward* Coriolis force acting on the golf ball, equal in magnitude to

$$F_c = 2\, m\, \Omega\, v\, \sin\varphi, \tag{9}$$

where φ is the latitude and m is the mass of the golf ball. This will cause the ball to slice the right. The effect is very slight, though. For example, given the rotation rate of the Earth $\Omega = 7.2921 \times 10^{-5}$ rad/s, the mass of the golf ball $m = 45$ g, a typical ball speed $v = 50$ m/s, and a latitude of $\varphi = 39°$, the Coriolis force only amounts to $F_c = 206.5\ \mu$N, or about 0.05% of the weight of the golf ball.

The Coriolis force is zero at the equator, and greater at higher latitudes. In the southern hemisphere, the Coriolis force will cause a slight hook of the ball to the left, rather than the slice it will experience in the northern hemisphere.

Weather

By Eq. (8), we can see that in the northern hemisphere, air currents moving northward are deflected to the east; eastward currents are deflected to the south; southward currents are deflected to the west; and westward currents are deflected to the north. If a low-pressure area forms in the atmosphere, then the pressure gradients will cause the air currents to flow toward the center of the area; but because of the Coriolis deflections, the result will be that the air currents will flow counter-clockwise, creating an air pattern called a *cyclone* around the low-pressure area. Similarly, in the southern hemisphere, cyclones will be air currents rotating clockwise.