Integration by Parts

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1 Integration by Parts

Integration by parts is a kind of product rule for integral calculus. Given two functions $u(x)$ and $v(x)$, then $du = u'(x)\,dx$ and $dv = v'(x)\,dx$. The formula for integration by parts is then

$$\int u\,dv = uv - \int v\,du \tag{1}$$

For definite integrals,

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du \tag{2}$$

For a given integral whose integrand is the product of two functions, how do we choose which part of the integrand will be u and which will be dv? A good rule of thumb is given by the so-called *ILATE rule*. Note the following list:

- Inverse trigonometric
- Logarithmic
- Algebraic
- Trigonometric
- Exponential

The ILATE rules states that you should *pick u from near the top of the list.* This is said to work about 90% of the time, but not always.[1]

[1] John W. Layman, Dept. of Mathematics, Virginia Tech, 1980.

Example 1

Given the integral

$$\int xe^{2x}\,dx \tag{3}$$

we see that x (algebraic) is closer to the top of the list than e^{2x} (exponential), so we pick $u = x$. From there we find $dv = e^{2x}dx$, $du = dx$, and $v = \frac{1}{2}e^{2x}$, and so the integral is

$$uv - \int v\,du = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$$

Example 2

Given the integral

$$\int \sqrt{x}\ln x\,dx \tag{4}$$

we see that $\ln x$ (logarithmic) is closer to the top of the list than $\sqrt{x}$ (algebraic), so we pick $u = \ln x$. From there we find $dv = \sqrt{x}dx$, $du = \frac{1}{x}dx$, and $v = \frac{2}{3}x^{3/2}$, and so the integral is

$$uv - \int v\,du = \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C$$

Example 3

Given the integral

$$\int x\sec^2 x\,dx \tag{5}$$

we see that x (algebraic) is closer to the top of the list than $\sec^2 x$ (trigonometric), so we pick $u = x$. From there we find $dv = \sec^2 xdx$, $du = dx$, and $v = \tan x$, and so the integral is

$$uv - \int v\,du = x\tan x + \ln\cos x + C$$