Rotating Bodies

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Translational vs. Rotational Motion

Many of the formulæ involving rotational motion are similar to the formulæ we saw in translational motion, and we can use the same methods for working with them. Each of the quantities we encountered in translational motion has a rotational counterpart, as shown in Table 1. (Time t is the same in both translational and rotational motion.)

Table 1. Translational and rotational quantities. This table shows several quantities related to translational motion, along with their counterparts in rotational motion and how the two are related.

Translational Motion		Rotational Motion		
Name	Symbol	Name	Symbol	Relationship
Position	x	Angle	θ	$\theta = s/r$
Velocity	v	Angular velocity	ω	$\omega = v_t/r$
Acceleration	a	Angular acceleration	α	$\alpha = a_t/r$
Mass	m	Moment of inertia	I	$I = \int r^2 dm$
Force	F	Torque	τ	$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$
Momentum	p	Angular momentum	L	$\mathbf{L} = \mathbf{r} \times \mathbf{p}$

(In the first three lines, s is arc length, and v_t and a_t are the tangential components of the velocity and acceleration, respectively.)

Many of the translational formulæ we've encountered so far have a similar formula in rotational motion. We can generally find these rotational formulæ by replacing the translational variables with the corresponding rotational variables from Table 1. Examples of such formulæ are shown in Table 2.

Table 2. Translational and rotational formulæ. This table shows a number of formulæ from translational mechanics, along with their rotational counterparts.

Description	Translational Motion	Rotational Motion
Velocity	$v = dx/dt$	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	$\alpha = d\omega/dt$
Constant acceleration	$x = \frac{1}{2}at^2 + v_0 t + x_0$	$\theta = \frac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$
" "	$v = at + v_0$	$\omega = \alpha t + \omega_0$
" "	$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\,\boldsymbol{\alpha} \cdot (\mathbf{r} - \mathbf{r_0})$
Newton's 2nd Law (const. mass)	$F = ma$	$\tau = I\alpha$
Newton's 2nd Law (general)	$F = dp/dt$	$\tau = dL/dt$
Momentum	$p = mv$	$L = I\omega$
Work	$W = Fx$	$W = \tau\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$
" "	$K = p^2/2m$	$K = L^2/2I$
Hooke's Law	$F = -kx$	$\tau = -\kappa\theta$
Potential energy (spring)	$U_s = \frac{1}{2}kx^2$	$U_s = \frac{1}{2}\kappa\theta^2$

Example Problems

Translational Problem

Consider the following translational problem: a body of mass $m = 3.0$ kg is initally at rest; then a force of $F = 5.0$ N is applied to it for time $t = 7.0$ seconds. What is the final velocity v of the body?

Solution. Given the force, we can find the acceleration; knowing the acceleration and time, we can find the velocity. The applicable equations are

$$F = ma \tag{1}$$
$$v = at + v_0. \tag{2}$$

Solving Eq. (1) for a and substituting into Eq. (2), we have

$$v = \left(\frac{F}{m}\right) t + v_0. \tag{3}$$

Substituting the given values of F, m, and t, and using $v_0 = 0$, we have

$$v = \left(\frac{5.0 \text{ N}}{3.0 \text{ kg}}\right) (7.0 \text{ s}), \tag{4}$$

or

$$\boxed{v = 11.67 \text{ m/s}} \tag{5}$$

Rotational Problem

Now consider the following similar rotational problem, which can be solved using the same method: a body of moment of inertia $I = 3.0$ kg m^2 is initally at rest (not rotating); then a torque of $\tau = 5.0$ N m is applied to it for time $t = 7.0$ seconds. What is the final angular velocity ω of the body?

Solution. Given the torque, we can find the angular acceleration; knowing the angular acceleration and time, we can find the angular velocity. The applicable equations are analogous to those used for the translational problem:

$$\tau = I\alpha \tag{6}$$

$$\omega = \alpha t + \omega_0. \tag{7}$$

Solving Eq. (6) for α and substituting into Eq. (7), we have

$$\omega = \left(\frac{\tau}{I}\right) t + \omega_0. \tag{8}$$

Substituting the given values of τ, I, and t, and using $\omega_0 = 0$, we have

$$\omega = \left(\frac{5.0 \text{ N m}}{3.0 \text{ kg m}^2}\right) (7.0 \text{ s}), \tag{9}$$

or

$$\boxed{\omega = 11.67 \text{ rad/s}} \tag{10}$$