Thermodynamics

D.G. Simpson, Ph.D.

Department of Physical Sciences and Engineering
Prince George's Community College
Largo, Maryland

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Foreword

After writing up class notes for General Physics I (calculus-based classical mechanics) and Introductory Physics II (algebra-based waves, acoustics, electricity and magnetism, optics, and modern physics) for courses at Prince George's Community College, I realized that the notes together covered most of the major areas of physics, with one important exception: thermodynamics. To make the set of notes complete, I decided to write up these notes on thermodynamics to complement the other two sets of notes. I've written them at the level of the General Physics (calculus-based) sequence.

D.G. Simpson, Ph.D.
Largo, Maryland
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Chapter 1

What is Physics?

Physics is the most fundamental of the sciences. Its goal is to learn how the Universe works at the most fundamental level—and to discover the basic laws by which it operates. *Theoretical physics* concentrates on developing the theory and mathematics of these laws, while *applied physics* focuses attention on the application of the principles of physics to practical problems. *Experimental physics* lies at the intersection of physics and engineering; experimental physicists have the theoretical knowledge of theoretical physicists, *and* they know how to build and work with scientific equipment.

Physics is divided into a number of sub-fields, and physicists are trained to have some expertise in all of them. This variety is what makes physics one of the most interesting of the sciences—and it makes people with physics training very versatile in their ability to do work in many different technical fields.

The major fields of physics are:

- *Classical mechanics* is the study the motion of bodies according to Newton's laws of motion.
- *Electricity and magnetism* are two closely related phenomena that are together considered a single field of physics.
- *Quantum mechanics* describes the peculiar motion of very small bodies (atomic sizes and smaller).
- *Optics* is the study of light.
- *Acoustics* is the study of sound.
- *Thermodynamics* and *statistical mechanics* are closely related fields that study the nature of heat. Thermodynamics is the subject of these notes.
- *Solid-state physics* is the study of solids—most often crystalline metals.
- *Plasma physics* is the study of plasmas (ionized gases).
- *Atomic, nuclear, and particle physics* study of the atom, the atomic nucleus, and the particles that make up the atom.
- *Relativity* includes Albert Einstein's theories of special and general relativity. *Special relativity* describes the motion of bodies moving at very high speeds (near the speed of light), while *general relativity* is Einstein's theory of gravity.

The fields of *cross-disciplinary physics* combine physics with other sciences. These include *astrophysics* (physics of astronomy), *geophysics* (physics of geology), *biophysics* (physics of biology), *chemical physics* (physics of chemistry), and *mathematical physics* (mathematical theories related to physics).

Besides acquiring a knowledge of physics for its own sake, the study of physics will give you a broad technical background and set of problem-solving skills that you can apply to wide variety of other fields. Some students of physics go on to study more advanced physics, while others find ways to apply their knowledge of physics to such diverse subjects as mathematics, engineering, biology, medicine, and finance.

Chapter 2

Units

The phenomena of Nature have been found to obey certain physical laws; one of the primary goals of physics research is to discover those laws. It has been known for several centuries that the laws of physics are appropriately expressed in the language of *mathematics*, so physics and mathematics have enjoyed a close connection for quite a long time.

In order to connect the physical world to the mathematical world, we need to make *measurements* of the real world. In making a measurement, we compare a physical quantity with some agreed-upon standard, and determine how many such standard units are present. For example, we have a precise definition of a unit of length called a *mile*, and have determined that there are about 92,000,000 such miles between the Earth and the Sun.

It is important that we have very precise definitions of physical units — not only for scientific use, but also for trade and commerce. In practice, we define a few *base units*, and derive other units from combinations of those base units. For example, if we define units for length and time, then we can define a unit for speed as the length divided by time (e.g. miles/hour).

How many base units do we need to define? There is no magic number; in fact it is possible to define a system of units using only *one* base unit (and this is in fact done for so-called *natural units*). For most systems of units, it is convenient to define base units for length, mass, and time; a base electrical unit may also be defined, along with a few lesser-used base units.

2.1 Systems of Units

Several different systems of units are in common use. For everyday civil use, most of the world uses *metric* units. The United Kingdom uses both metric units and an *imperial* system. Here in the United States, *U.S. customary units* are most common for everyday use.[1]

There are actually several "metric" systems in use. They can be broadly grouped into two categories: those that use the meter, kilogram, and second as base units (MKS systems), and those that use the centimeter, gram, and second as base units (CGS systems). There is only one MKS system, called *SI units*. We will mostly use SI units in this course.

[1] In the mid-1970s the U.S. government attempted to switch the United States to the metric system, but the idea was abandoned after strong public opposition. One remnant from that era is the two-liter bottle of soda pop.

2.2 SI Units

SI units (which stands for Système International d'unités) are based on the *meter* as the base unit of length, the *kilogram* as the base unit of mass, and the *second* as the base unit of time. SI units also define four other base units (the *ampere*, *kelvin*, *candela*, and *mole*, to be described later). Any physical quantity that can be measured can be expressed in terms of these base units or some combination of them. SI units are summarized in Appendix E.

Length (Meter)

The SI base unit of length, the *meter* (m), has been re-defined more times than any other unit, due to the need for increasing accuracy. Originally (1793) the meter was defined to be $1/10{,}000{,}000$ the distance from the North Pole to the equator, along a line going through Paris.[2] Then, in 1889, the meter was re-defined to be the distance between two lines engraved on a prototype meter bar kept in Paris. Then in 1960 it was re-defined again: the meter was defined as the distance of 1,650,763.73 wavelengths of the orange-red emission line in the krypton-86 atomic spectrum. Still more stringent accuracy requirements led to the the current definition of the meter, which was implemented in 1983: the meter is now defined to be the distance light in vacuum travels in $1/299{,}792{,}458$ second. Because of this definition, the speed of light is now *exactly* 299,792,458 m/s.

U.S. Customary units are legally defined in terms of metric equivalents. For length, the *foot* (ft) is defined to be exactly 0.3048 meter.

Mass (Kilogram)

Originally the *kilogram* (kg) was defined to be the mass of 1 liter (0.001 m^3) of water. The need for more accuracy required the kilogram to be re-defined to be the mass of a standard mass called the *International Prototype Kilogram* (IPK, frequently designated by the Gothic letter $\mathfrak{K}$), which is kept in a vault at the Bureau International des Poids et Mesures (BIPM) in Paris. The kilogram is the only base unit still defined in terms of a prototype, rather than in terms of an experiment that can be duplicated in the laboratory.

The International Prototype Kilogram is a small cylinder of platinum-iridium alloy (90% platinum), about the size of a golf ball. In 1884, a set of 40 duplicates of the IPK was made; each country that requested one got one of these duplicates. The United States received two of these: the duplicate called K20 arrived here in 1890, and has been the standard of mass for the U.S. ever since. The second copy, called K4, arrived later that same year, and is used as a constancy check on K20. Finally, in 1996 the U.S. got a third standard called K79; this is used for mass stability studies. These duplicates are kept at the National Institutes of Standards and Technology (NIST) in Gaithersburg, Maryland. They are kept under very controlled conditions under several layers of glass bell jars and are periodically cleaned. From time to time they are returned to the BIPM in Paris for re-calibration. For reasons not entirely understood, very careful calibration measurements show that the masses of the duplicates do not stay exactly constant. Because of this, physicists are considering re-defining the kilogram sometime in the next few years.

Another common metric (but non-SI) unit of mass is the *metric ton*, which is 1000 kg (a little over 1 short ton).

In U.S. customary units, the *pound-mass* (lbm) is defined to be exactly 0.45359237 kg.

Mass vs. Weight

Mass is not the same thing as *weight*, so it's important not to confuse the two. The *mass* of a body is a measure of the total amount of matter it contains; the *weight* of a body is the gravitational force on it due to the Earth's gravity. At the surface of the Earth, mass m and weight W are proportional to each other:

[2] If you remember this original definition, then you can remember the circumference of the Earth: about $40{,}000{,}000$ meters.

$$W = mg, \tag{2.1}$$

where g is the acceleration due to the Earth's gravity, equal to 9.80 m/s^2. Remember: mass is mass, and is measured in kilograms; weight is a force, and is measured in force units of *newtons*.

Time (Second)

Originally the base SI unit of time, the *second* (s), was defined to be $1/60$ of $1/60$ of $1/24$ of the length of a day, so that 60 seconds = 1 minute, 60 minutes = 1 hour, and 24 hours = 1 day. High-precision time measurements have shown that the Earth's rotation rate has short-term irregularities, along with a long-term slowing due to tidal forces. So for a more accurate definition, in 1967 the second was re-defined to be based on a definition using atomic clocks. The second is now defined to be the time required for 9,192,631,770 oscillations of a certain type of radiation emitted from a cesium-133 atom.

Although officially the symbol for the second is "s", you will also often see people use "sec" to avoid confusing lowercase "s" with the number "5".

The Ampere, Kelvin, and Candela

For this course, most quantities will be defined entirely in terms of meters, kilograms, and seconds. There are four other SI base units, though: the *ampere* (A) (the base unit of electric current); the *kelvin* (K) (the base unit of temperature); the *candela* (cd) (the base unit of luminous intensity, or light brightness); and the *mole* (mol) (the base unit of amount of substance).

Amount of Substance (Mole)

Since we may have a use for the mole in this course, let's look at its definition in detail. The simplest way to think of it is as the name for a number. Just as "thousand" means 1,000, "million" means 1,000,000, and "billion" means 1,000,000,000, in the same way "mole" refers to the number 602,214,129,000,000,000,000,000, or $6.02214129 \times 10^{23}$. You could have a mole of grains of sand or a mole of Volkswagens, but most often the mole is used to count atoms or molecules. There is a reason this number is particularly useful: since each nucleon (proton and neutron) in an atomic nucleus has an average mass of $1.660538921 \times 10^{-24}$ grams (called an *atomic mass unit*, or amu), then there are $1/(1.660538921 \times 10^{-24})$, or $6.02214129 \times 10^{23}$ nucleons per gram. In other words, one mole of nucleons has a mass of 1 gram. Therefore, if A is the atomic weight of an atom, then A moles of nucleons has a mass of A grams. But A moles of nucleons is the same as 1 mole of atoms, so *one mole of atoms has a mass (in grams) equal to the atomic weight*. In other words,

$$\text{moles of atoms} = \frac{\text{grams}}{\text{atomic weight}} \tag{2.2}$$

Similarly, when counting molecules,

$$\text{moles of molecules} = \frac{\text{grams}}{\text{molecular weight}} \tag{2.3}$$

In short, the mole is useful when you need to convert between the mass of a material and the number of atoms or molecules it contains.

It's important to be clear about what exactly you're counting (atoms or molecules) when using moles. It doesn't really make sense to talk about "a mole of oxygen", any more than it would be to talk about "100 of oxygen". It's either a "mole of oxygen atoms" or a "mole of oxygen molecules".[3]

Interesting fact: there is about ½ mole of stars in the observable Universe.

[3] Sometimes chemists will refer to a "mole of oxygen" when it's understood whether the oxygen in question is in the atomic (O) or molecular (O_2) state.

SI Derived Units

In addition to the seven base units (m, kg, s, A, K, cd, mol), there are a number of so-called *SI derived units* with special names. We'll introduce these as needed, but a summary of all of them is shown in Appendix E (Table E-2). These are just combinations of base units that occur often enough that it's convenient to give them special names.

Plane Angle (Radian)

One derived SI unit that we will encounter frequently is the SI unit of plane angle. Plane angles are commonly measured in one of two units: *degrees* or *radians*.[4] You're probably familiar with degrees already: one full circle is 360°, a semicircle is 180°, and a right angle is 90°.

The SI unit of plane angle is the *radian*, which is defined to be that plane angle whose arc length is equal to its radius. This means that a full circle is 2π radians, a semicircle is π radians, and a right angle is $\pi/2$ radians. To convert between degrees and radians, then, we have:

$$\text{degrees} = \text{radians} \times \frac{180}{\pi} \tag{2.4}$$

and

$$\text{radians} = \text{degrees} \times \frac{\pi}{180} \tag{2.5}$$

The easy way to remember these formulæ is to think in terms of units: 180 has units of degrees and π has units of radians, so in the first equation units of radians cancel on the right-hand side to leave degrees, and in the second equation units of degrees cancel on the right-hand side to leave radians.

Occasionally you will see a formula that involves a "bare" angle that is not the argument of a trigonometric function like the sine, cosine, or tangent. In such cases it is understood that the angle must be *in radians*. For example, the radius of a circle r, angle θ, and arc length s are related by

$$s = r\theta, \tag{2.6}$$

where it is understood that θ is in radians.

See Appendix K for a further discussion of plane and solid angles.

SI Prefixes

It's often convenient to define both large and small units that measure the same thing. For example, in English units, it's convenient to measure small lengths in inches and large lengths in miles.

In SI units, larger and smaller units are defined in a systematic way by the use of *prefixes* to the SI base or derived units. For example, the base SI unit of length is the meter (m), but small lengths may also be measured in centimeters (cm, 0.01 m), and large lengths may be measured in kilometers (km, 1000 m). Table E-3 in Appendix E shows all the SI prefixes and the powers of 10 they represent. You should *memorize* the powers of 10 for all the SI prefixes in this table.

To use the SI prefixes, simply add the prefix to the front of the name of the SI base or derived unit. The symbol for the prefixed unit is the symbol for the prefix written in front of the symbol for the unit. For example, kilometer (km) = 10^3 meter, microsecond (μs) = 10^{-6} s. But put the prefix on the *gram* (g), *not* the kilogram: for example, 1 microgram (μg) = 10^{-6} g. For historical reasons, the kilogram is the only SI base or derived unit with a prefix.

[4] A third unit implemented in many calculators is the *grad*: a right angle is 100 grads and a full circle is 400 grads. You may encounter grads in some older literature, such as Laplace's *Mécanique Céleste*. Almost nobody uses grads today, though.

2.3 CGS Systems of Units

In some fields of physics (e.g. solid-state physics, plasma physics, and astrophysics), it has been customary to use CGS units rather than SI units, so you may encounter them occasionally. There are several different CGS systems in use: *electrostatic*, *electromagnetic*, *Gaussian*, and *Heaviside-Lorentz* units. These systems differ in how they define their electric and magnetic units. Unlike SI units, none of these CGS systems defines a base electrical unit, so electric and magnetic units are all derived units. The most common of these CGS systems is Gaussian units, which are summarized in Appendix F.

SI prefixes are used with CGS units in the same way they're used with SI units.

2.4 British Engineering Units

Another system of units that is common in some fields of engineering is *British engineering units*. In this system, the base unit of length is the foot (ft), and the base unit of time is the second (s). There is no base unit of mass; instead, one uses a base unit of force called the *pound-force* (lbf). Mass in British engineering units is measured units of *slugs*, where 1 slug has a weight of 32.17404855 lbf.

A related unit of mass (not part of the British engineering system) is called the pound-mass (lbm). At the surface of the Earth, a mass of 1 lbm has a weight of 1 lbf, so sometimes the two are loosely used interchangeably and called the *pound* (lb), as we do every day when we speak of weights in pounds.

SI prefixes are not used in the British engineering system.

2.5 Units as an Error-Checking Technique

Checking units can be used as an important error-checking technique called *dimensional analysis*. If you derive an equation and find that the units don't work out properly, then you can be certain you made a mistake somewhere. If the units are correct, it doesn't necessarily mean your derivation is correct (since you could be off by a factor of 2, for example), but it does give you some confidence that you at least haven't made a units error. So checking units doesn't tell you for certain whether or not you've made a mistake, but it does help.

Here are some basic principles to keep in mind when working with units:

1. Units on both sides of an equation must match.
2. When adding or subtracting two quantities, they must have the same units.
3. The argument for functions like sin, cos, tan, $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, log, and exp must be dimensionless.
4. When checking units, radians and steradians can be considered dimensionless.

Sometimes it's not clear whether or not the units match on both sides of the equation, for example when both sides involve derived SI units. In that case, it may be useful to break all the derived units down in terms of base SI units (m, kg, s, A, K, mol, cd). Table E-2 in Appendix E shows each of the derived SI units broken down in terms of base SI units.

2.6 Unit Conversions

It is very common to have to work with quantities that are given in units other than the units you'd like to work with. Converting from one set of units to another involves a straightforward, virtually foolproof technique that's very simple to double-check. We'll illustrate the method here with some examples.

Appendix J gives a number of important conversion factors. More conversion factors are available from sources such as the *CRC Handbook of Chemistry and Physics*.

1. Write down the unit conversion factor as a ratio, and fill in the units in the numerator and denominator so that the units cancel out as needed.

2. Now fill in the numbers so that the numerator and denominator contain the same length, time, etc. (This is because you want each factor to be a multiplication by 1, so that you don't change the quantity—only its units.)

Simple Conversions

A simple unit conversion involves only one conversion factor. The method for doing the conversion is best illustrated with an example.

Example. Convert 7 feet to inches.
Solution. First write down the unit conversion factor as a ratio, filling in the units as needed:

$$(7 \text{ ft}) \times \frac{\text{in}}{\text{ft}} \tag{2.7}$$

Notice that the units of feet cancel out, leaving units of inches. The next step is to fill in numbers so that the same length is in the numerator and denominator:

$$(7 \text{ ft}) \times \frac{12 \text{ in}}{1 \text{ ft}} \tag{2.8}$$

Now do the arithmetic:

$$(7 \text{ ft}) \times \frac{12 \text{ in}}{1 \text{ ft}} = 84 \text{ inches.} \tag{2.9}$$

More Complex Conversions

More complex conversions may involve more than one conversion factor. You'll need to think about what conversion factors you know, then put together a chain of them to get to the units you want.

Example. Convert 60 miles per hour to feet per second.
Solution. First, write down a chain of conversion factor ratios, filling in units so that they cancel out correctly:

$$60 \; \frac{\text{mile}}{\text{hr}} \times \frac{\text{ft}}{\text{mile}} \times \frac{\text{hr}}{\text{sec}} \tag{2.10}$$

Units cancel out to leave ft/sec. Now fill in the numbers, putting the same length in the numerator and denominator in the first factor, and the same time in the numerator and denominator in the second factor:

$$60 \; \frac{\text{mile}}{\text{hr}} \times \frac{5280 \text{ ft}}{1 \text{ mile}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} \tag{2.11}$$

Finally, do the arithmetic:

$$60 \; \frac{\text{mile}}{\text{hr}} \times \frac{5280 \text{ ft}}{1 \text{ mile}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} = 88 \; \frac{\text{ft}}{\text{sec}} \tag{2.12}$$

Example. Convert 250,000 furlongs per fortnight to meters per second.

Solution. We don't know how to convert furlongs per fortnight directly to meters per second, so we'll have to come up with a chain of conversion factors to do the conversion. We *do* know how to convert: furlongs to miles, miles to kilometers, kilometers to meters, fortnights to weeks, weeks to days, days to hours, hours to minutes, and minutes to seconds. So we start by writing conversion factor ratios, putting units where they need to be so that the result will have the desired target units (m/s):

$$250{,}000\ \frac{\text{furlong}}{\text{fortnight}} \times \frac{\text{mile}}{\text{furlong}} \times \frac{\text{km}}{\text{mile}} \times \frac{\text{m}}{\text{km}} \times \frac{\text{fortnight}}{\text{week}} \times \frac{\text{week}}{\text{day}} \times \frac{\text{day}}{\text{hr}} \times \frac{\text{hr}}{\text{min}} \times \frac{\text{min}}{\text{sec}}$$

If you check the units here, you'll see that almost everything cancels out; the only units left are m/s, which is what we want to convert to. Now fill in the numbers: we want to put either the same length or the same time in both the numerator and denominator:

$$250{,}000\ \frac{\text{furlong}}{\text{fortnight}} \times \frac{1\ \text{mile}}{8\ \text{furlongs}} \times \frac{1.609344\ \text{km}}{1\ \text{mile}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{fortnight}}{2\ \text{weeks}} \times \frac{1\ \text{week}}{7\ \text{days}} \times \frac{1\ \text{day}}{24\ \text{hr}} \times \frac{1\ \text{hr}}{60\ \text{min}} \times \frac{1\ \text{min}}{60\ \text{sec}}$$
$$= 41.58\ \text{m/s}$$

Conversions Involving Powers

Occasionally we need to do something like convert an area or volume when we know only the length conversion factor.

Example. Convert 2000 cubic feet to gallons.

Solution. Let's think about what conversion factors we know. We know the conversion factor between gallons and cubic inches. We don't know the conversion factor between cubic feet and cubic inches, but we can convert between feet and inches. The conversion factors will look like this:

$$2000\ \text{ft}^3 \times \left(\frac{\text{in}}{\text{ft}}\right)^3 \times \frac{\text{gal}}{\text{in}^3} \tag{2.13}$$

With these units, the whole expression reduces to units of gallons. Now fill in the same length in the numerator and denominator of the first factor, and the same volume in the numerator and denominator of the second factor:

$$2000\ \text{ft}^3 \times \left(\frac{12\ \text{in}}{1\ \text{ft}}\right)^3 \times \frac{1\ \text{gal}}{231\ \text{in}^3} \tag{2.14}$$

Now do the arithmetic:

$$2000\ \text{ft}^3 \times \left(\frac{12\ \text{in}}{1\ \text{ft}}\right)^3 \times \frac{1\ \text{gal}}{231\ \text{in}^3} = 14{,}961\ \text{gallons} \tag{2.15}$$

2.7 Odds and Ends

We'll end this chapter with a few miscellaneous notes about SI units:

- In a few special cases, we customarily drop the ending vowel of a prefix when combining with a unit that begins with a vowel: it's *megohm* (*not* "megaohm"); *kilohm* (*not* "kiloohm"); and *hectare* (*not* "hectoare"). In all other cases, keep both vowels (e.g. *microohm*, *kiloare*, etc.). There's no particular reason for this—it's just customary.

- Sometimes in electronics work the SI prefix symbol may be used in place of the decimal point. For example, 24.9 MΩ may be written "24M9". This saves space on electronic diagrams and when printing values on electronic components, and also avoids problems with the decimal point being nearly invisible when the print is tiny. This is unofficial use, and is only encountered in electronics.

- One sometimes encounters older metric units of length called the *micron* (μ, now properly called the *micrometer*, 10^{-6} meter) and the *millimicron* (mμ, now properly called the *nanometer*, 10^{-9} meter). The micron and millimicron are now obsolete.

- In computer work, the SI prefixes are often used with units of bytes, but may refer to powers of 2 that are near the SI values. For example, the term "1 kB" may mean 1000 bytes, or it may mean $2^{10} = 1024$ bytes. Similarly, a 100 GB hard drive may have a capacity of 100,000,000,000 bytes, or it may mean $100 \times 2^{30} = 107{,}374{,}182{,}400$ bytes. To help resolve these ambiguities, a set of *binary prefixes* has been introduced (Table E-4 of Appendix E). These prefixes have not yet entirely caught on in the computing industry, though.

Chapter 3

Problem-Solving Strategies

Much of this course will focus on developing your ability to solve physics problems. If you enjoy solving puzzles, you'll find solving physics problems is similar in many ways. Here we'll look at a few general tips on how to approach solving problems.

- At the beginning of the problem, immediately convert the units of all the quantities you're given to base SI units. In other words, convert all lengths to meters, all masses to kilograms, all times to seconds, etc.: all quantities should be in un-prefixed SI units, except for masses in kilograms. When you do this, you're guaranteed that the final result will also be in base SI units, and this will minimize your problems with units. As you gain more experience in problem solving, you'll sometimes see shortcuts that let you get around this suggestion, but for now converting all units to base SI units is the safest approach.

- Look at the information you're given, and what you're being asked to find. Then think about what equations you know that might let you get from what you're given to what you're trying to find.

- Be sure you understand under what conditions each equation is valid. For example, we'll shortly see a set of equations that are derived by assuming constant acceleration. It would be inappropriate to use those equations for a mass on a spring, since the acceleration of a mass under a spring force is *not* constant. For each equation you're using, you should be clear what each variable represents, and under what conditions the equation is valid.

- As a general rule, it's best to derive an algebraic expression for the solution to a problem first, then substitute numbers to compute a numerical answer as the very last step. This approach has a number of advantages: it allows you to check units in your algebraic expression, helps minimize roundoff error, and allows you to easily repeat the calculation for different numbers if needed.

- If you've derived an algebraic equation, *check the units* of your answer. Make sure your equation has the correct units, and doesn't do something like add quantities with different units.

- If you've derived an algebraic equation, you can check that it has the proper behavior for extreme values of the variables. For example, does the answer make sense if time $t \to \infty$? If the equation contains an angle, does it reduce to a sensible answer when the angle is $0°$ or $90°$?

- Check your answer for reasonableness—don't just write down whatever your calculator says. For example, suppose you're computing the speed of a pendulum bob in the laboratory, and find the answer is 14,000 miles per hour. That doesn't seem reasonable, so you should go back and check your work.

- You can avoid rounding errors by carrying as many significant digits as possible throughout your calculations; don't round off until you get to the final result.
- Write down a reasonable number of significant digits in the final answer—don't write down all the digits in your calculator's display. Nor should you round too much and use too few significant digits. There are rules for determining the correct number of significant digits, but for most problems in this course, 3 or 4 significant digits will be about right.
- Don't forget to put the correct units on the final answer! You will have points deducted for forgetting to do this.
- The best way to get good at problem solving (and to prepare for exams for this course) is *practice*—practice working as many problems as you have time for. Working physics problems is a skill much like learning to play a sport or musical instrument. You can't learn by watching someone else do it—you can only learn it by doing it yourself.

Chapter 4

Temperature

4.1 Thermodynamics

Thermodynamics is the study of heat, and the transfer of energy between bodies due differences in temperature.

4.2 Temperature

We have an intuitive sense of temperature from encountering it in everyday life: when a body (or the air) has a high temperature, it "feels" hotter; when the temperature is low, it feels colder. This intuitive sense of temperature breaks down in some situations (in a near vacuum, for example), and we will require a more precise scientific definition.

Technically, the *temperature* of a body is a measure of the average kinetic energy of the particles making up the body. Temperature therefore *can* be expressed in energy units, but it is more commonly expressed on a *temperature scale*, as described in the following section.

4.3 Temperature Scales

Several scales for measuring temperature are in common use. For everyday civil use in the United States, the most common temperature scale is the *Fahrenheit* scale, in which temperature is measured in *degrees Fahrenheit* (°F).[1] On the Fahrenheit scale, water freezes at 32°F and boils at 212°F, and so the interval between these two points is 180°. A normal comfortable (slightly cold) "room temperature" is about 68°F, and nominal human body temperature is 98.6°F.

Throughout much of the rest of the world, the common temperature scale in civil use is the *Celsius* scale, in which temperature is measured in *degrees Celsius* (°C).[2] On the Celsius scale, water freezes at 0°C and boils at 100°C; room temperature is 20°C, and nominal human body temperature is 37°C.

The Fahrenheit and Celsius scales are related by the equations

$$^\circ\mathrm{C} = \tfrac{5}{9}(^\circ\mathrm{F} - 32) \tag{4.1}$$

$$^\circ\mathrm{F} = \tfrac{9}{5}(^\circ\mathrm{C}) + 32 \tag{4.2}$$

[1] Named for physicist Daniel Fahrenheit.

[2] Named for the Swedish astronomer Anders Celsius. The Celsius scale is also known as the *centigrade* scale.

It is easy to show that the Fahrenheit and Celsius scales are equal at one point: $-40°\text{F} = -40°\text{C}$, which happens to be near the freezing point of mercury.

In scientific and engineering work, one often uses *absolute* temperature scales, in which 0° is set at the lowest possible temperature, called *absolute zero* (described in the following section). One such absolute temperature scale is the *Rankine* scale,[3] in which temperature is measured in *degrees Rankine* (°R). Intervals of 1° are the same on both the Fahrenheit and Rankine scales; the two scales differ only by the location of the 0° point. Since absolute zero is −459.67°F, the Fahrenheit and Rankine scales are related by

$$°\text{R} = °\text{F} + 459.67 \tag{4.3}$$

On the Rankine scale, water freezes at 491.67°R and boils at 671.67°R; room temperature is 527.67°R, and nominal human body temperature is 558.27°R.

The SI unit for temperature is an absolute temperature scale called the *Kelvin scale*, in which temperature is measured in *kelvins*.[4] Intervals of 1° are the same on both the Celsius and Kelvin scales, and the two scales differ only by the location of the 0° point. Since absolute zero is −273.15°C, the Celsius and Kelvin scales are related by

$$\text{K} = °\text{C} + 273.15 \tag{4.4}$$

On the Kelvin scale, water freezes at 273.15 K and boils at 373.15 K; room temperature is 293.15 K, and nominal body temperature is 310.15 K.

4.4 Absolute Zero

As mentioned earlier, the temperature of a body is a measure of the average energy per molecule of the body. As the body is cooled more and more, this average energy will become less and less. Because of effects due to quantum mechanics, this average energy cannot reach zero, but there is a minimum-energy limit beyond which the body cannot be cooled any further. This minimum-energy temperature is called *absolute zero*, and is the lowest temperature to which any body can be cooled. Absolute zero is equal to:

- 0 K
- 0 °R
- −273.15 °C
- −459.67 °F

4.5 "Absolute Hot"

If absolute zero is the coldest possible temperature, it is natural to ask: is there a *hottest* possible temperature? The answer is: nobody really knows.

This hypothetical highest temperature, if it exists, has been named "*absolute hot*." Nobody knows whether or not there is an "absolute hot," but we *can* say that our current best theories of physics break down above energies that correspond to the *Planck temperature*

$$T_P = \sqrt{\frac{\hbar c^5}{G k_B^2}} = 1.417 \times 10^{32}\ \text{K}, \tag{4.5}$$

[3] Named for Scottish engineer and physicist William Rankine.

[4] Named for William Thompson, Lord Kelvin. Note that the Kelvin scale does *not* use the degree symbol (°).

or 141.7 *nonillion* kelvins. (Here $\hbar$ is Planck's constant divided by 2π, c is the speed of light in vacuum, G is Newton's gravitational constant, and k_B is Boltzmann's constant.) According to current cosmological models, the Universe was at this temperature just 5.4×10^{-44} seconds[5] after the Big Bang—the initial "explosion" in which the Universe was created. At this time, the entire Universe was only about 1.6×10^{-35} meters[6] in size.

4.6 Temperature of Space

What is the temperature of (outer) space? Remember that temperature is a measure of the average energy of the molecules that make up a body. When we speak of the temperature in a room or outdoors, we're referring to the temperature of the *air*. But space is essentially a vacuum, and it's meaningless to talk about the "temperature" of a vacuum—there's nothing in the vacuum whose energy we can measure. When people talk about the "temperature" in space, they may be talking about the temperature of the outer surface of the Space Station, or of the outer surface of an astronaut's space suit, or of the temperature of the soil on the surface of the Moon, depending on the context.

There is a sense, though, in which space *does* have a temperature. All of space is filled with microwave radiation, whose peak intensity is at a wavelength of about 1 mm. This radiation, which is blackbody radiation left over from the Big Bang, corresponds to a temperature of 2.73 K, and is called the *cosmic microwave background radiation*. Blackbody radiation will be discussed later.

4.7 Thermometry

Thermometry is the measurement of temperature. The most common method for measuring temperature in everyday life is with a *thermometer*. A typical thermometer consists of a tube of glass with a narrow channel inside, into which has been placed a quantity of mercury or colored alcohol. As the temperature increases, the liquid in the channel expands (see Chapter 5), causing it to rise upward. A scale is marked on the glass tube, calibrated on the Fahrenheit or Celsius temperature scales. Reading the top edge of the level of the liquid against the scale gives the temperature.

Of course, this type of thermometer will not work above the boiling point of the liquid, or below its freezing point (or above the melting point of the glass tube). Several other methods are available for measuring temperature outside this range:

- A *thermocouple* consists of two dissimilar metals in contact with each other. When the metals are in contact, there is a voltage produced, and this voltage is dependent on the temperature of the metals. My measuring the voltage across an appropriately calibrated thermocouple, one may measure the temperature.
- A *thermistor* is a resistor especially designed to have a resistance that is particularly sensitive to temperature. By measuring the resistance of a calibrated thermistor, one may measure the temperature.
- An *optical pyrometer* can be used to measure the temperature of objects that are hot enough to glow, such as lamp filaments. In this device, one compares the color of a body with the color of a calibrated lamp filament; when the colors match, the body and filament are at the same temperature. This allows one to determine the temperature of the body as long (as the lamp filament is calibrated).

[5] This is known as the *Planck time*.
[6] This is known as the *Planck length*.

Chapter 5

Thermal Expansion

5.1 Linear Expansion

Solid bodies generally expand (or sometimes contract) with increasing temperature, a phenomenon called *thermal expansion*. For a one-dimensional body (such as a rod), the change in length is found to be proportional to the temperature change. If the rod has an initial length L_0 and has its temperature increased by an amount ΔT, the rod's length with change by an amount

$$\Delta L = \alpha L_0 \Delta T, \tag{5.1}$$

where α is a constant called the *coefficient of linear expansion*, and depends on the material. If the length of the rod *after* the expansion is L, then we can write $\Delta L = L - L_0$, and Eq. (5.1) can be written in the form

$$L = L_0(1 + \alpha)\Delta T. \tag{5.2}$$

5.2 Surface Expansion

If a two-dimensional body (a sheet of metal, for example) of initial area A_0 is subject to a temperature change ΔT, then its new area A will be given by

$$\Delta A = \gamma A_0 \Delta T, \tag{5.3}$$

where $\gamma = 2\alpha$ is the *coefficient of surface expansion*. Since $\Delta A = A - A_0$, we can write Eq. (5.3) as

$$A = A_0(1 + \gamma)\Delta T \tag{5.4}$$

5.3 Volume Expansion

If a three-dimensional body (a volume of metal or liquid, for example) of initial volume V_0 is subject to a temperature change ΔT, then its new volume V will be given by

$$\Delta V = \beta V_0 \Delta T \tag{5.5}$$

where $\beta = 3\alpha$ is the *coefficient of volume expansion*. Since $\Delta V = V - V_0$, we can write Eq. (5.5) as

$$V = V_0(1 + \beta)\Delta T \tag{5.6}$$

Chapter 6

Heat

6.1 Energy Units

6.2 Heat Capacity

6.3 Calorimetry

6.4 Mechanical Equivalent of Heat

Chapter 7

Phases of Matter

7.1 Solid

7.2 Liquid

7.3 Gas

7.4 Plasma

7.5 Freezing and Melting

7.6 Vaporization and Condensation

7.7 Sublimation and Deposition

7.8 Water

7.9 Ice

Chapter 8

Heat Transfer

8.1 Conduction

8.2 Convection

8.3 Radiation

Chapter 9

Blackbody Radiation

9.1 Wein's Law

Chapter 10

Entropy

Chapter 11

The Laws of Thermodynamics

Chapter 12

Pressure

12.1 Units

Chapter 13

Gas Laws

13.1 Boyle's Law

$$P \propto \frac{1}{V} \tag{13.1}$$

13.2 Gay-Lussac's Law

$$P \propto T \tag{13.2}$$

13.3 Charles's Law

$$V \propto T \tag{13.3}$$

13.4 Ideal Gas Law

Combined gas law:

$$PV \propto T \tag{13.4}$$

$$PV = nRT \tag{13.5}$$

$$PV = Nk_BT \tag{13.6}$$

13.5 Van der Waals Equation

Chapter 14

Kinetic Theory of Gases

14.1 The Equipartition Theorem

14.2 The Maxwell-Boltzmann Distribution

Chapter 15

Heat Engines

Chapter 16

Thermodynamic Potentials

16.1 Internal Energy

16.2 Enthalpy

16.3 Gibbs Free Energy

16.4 Helmholtz Free Energy

16.5 Grand Potential

16.6 Chemical Potential

Chapter 17

Partial Derivatives

The equations of Lagrangian and Hamiltonian mechanics are expressed in the language of partial differential equations. We will leave the methods for solving such equations to a more advanced course, but we can still write down the equations and explore some of their consequences. First, in order to understand these equations, we'll first need to understand the concept of *partial derivatives*.

17.1 First Partial Derivatives

You've already learned in a calculus course how to take the derivative of a function of one variable. For example, if

$$f(x) = 3x^2 + 7x^5 \tag{17.1}$$

then

$$\frac{df}{dx} = 6x + 35x^4. \tag{17.2}$$

But what if f is a function of more that one variable? For example, if

$$f(x, y) = 5x^3y^5 + 4y^2 - 7xy^6 \tag{17.3}$$

then how do we take the derivative of f? In this case, there are *two* possible first derivatives: one with respect to x, and one with respect to y. These are called *partial derivatives*, and are indicated using the "backward-6" symbol ∂ in place of the symbol d used for ordinary derivatives.

To compute a partial derivative with respect to x, you simply treat all variables except x as constants. Similarly, for the partial derivative with respect to y, you treat all variables except y as constants. For example, if $g(x, y) = 3x^4y^7$, then the partial derivative of g with respect to x is $\partial g/\partial x = 12x^3y^7$, since both 3 and y^7 are considered constants with respect to x.

As another example, the partial derivatives of Eq. (17.3) are

$$\frac{\partial f}{\partial x} = 15x^2y^5 - 7y^6 \tag{17.4}$$

$$\frac{\partial f}{\partial y} = 25x^3y^4 + 8y - 42xy^5 \tag{17.5}$$

Notice that in Eq. (17.4), the derivative of the term $4y^2$ with respect to x is 0, since $4y^2$ is treated as a constant.

17.2 Higher-Order Partial Derivatives

It is similarly possible to take higher-order partial derivatives. For a function of two variables $f(x, y)$, there are *three* possible second derivatives:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right); \qquad \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right); \qquad \text{and} \qquad \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right). \tag{17.6}$$

In the second case, the order of differentiation doesn't matter: $\partial^2 f/(\partial x \partial y) \equiv \partial^2 f/(\partial y \partial x)$. This property is known as *Clairaut's theorem*.

For example, suppose $f(x, y)$ is as given by Eq. (17.3). Then the second partial derivatives of f are found by taking partial derivatives of Eqs. (17.4) and (17.5):

$$\frac{\partial^2 f}{\partial x^2} = 30xy^5 \tag{17.7}$$

$$\frac{\partial^2 f}{\partial x \partial y} = 75x^2y^4 - 42y^5 \tag{17.8}$$

$$\frac{\partial^2 f}{\partial y^2} = 100x^3y^3 + 8 - 210xy^4 \tag{17.9}$$

Chapter 18

Maxwell Relations

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V \tag{18.1}$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P \tag{18.2}$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \tag{18.3}$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T \tag{18.4}$$

Chapter 19

Statistical Mechanics

Appendices

Appendix A

Further Reading

General

- *The Feynman Lectures on Physics* (Definitive Edition; 3 vol.) by Richard P. Feynman, Robert B. Leighton, and Matthew Sands (Addison-Wesley, Reading, Mass., 2006). This classic work is well known to all students of physics. These lectures were presented by Nobel laureate Richard Feynman to his physics class at the California Institute of Technology in the 1960s, and are considered a masterpiece of physics exposition by one if its greatest teachers. (The the audio for these lectures is also available on CD, in 20 volumes.)

- *Thinking Physics* (3rd ed.) by Lewis Carroll Epstein (Insight Press, San Francisco, 2009). A very nice collection of thought-provoking physics puzzles.

Appendix B

Greek Alphabet

Table B-1. The Greek alphabet.

Letter	Name
A α	Alpha
B β	Beta
Γ γ	Gamma
Δ δ	Delta
E ε	Epsilon
Z ζ	Zeta
H η	Eta
Θ θ	Theta
I ι	Iota
K κ	Kappa
Λ λ	Lambda
M μ	Mu
N ν	Nu
Ξ ξ	Xi
O o	Omicron
Π π	Pi
P ρ	Rho
Σ σ	Sigma
T τ	Tau
Υ υ	Upsilon
Φ φ	Phi
X χ	Chi
Ψ ψ	Psi
Ω ω	Omega

(Alternate forms: $ϐ = \beta$, $\partial = \delta$, $\epsilon = \varepsilon$, $\vartheta = \theta$, $\varkappa = \kappa$, $\varpi = \pi$, $\varrho = \rho$, $\varsigma = \sigma$, $\phi = \varphi$.)

Appendix C

Trigonometric Identities

Basic Formulæ

$$\sin^2\theta + \cos^2\theta \equiv 1$$
$$\sec^2\theta \equiv 1 + \tan^2\theta$$
$$\csc^2\theta \equiv 1 + \cot^2\theta$$

Angle Addition Formulæ

$$\sin(\alpha \pm \beta) \equiv \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) \equiv \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) \equiv \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Double-Angle Formulæ

$$\sin 2\theta \equiv 2\sin\theta\cos\theta$$
$$\cos 2\theta \equiv \cos^2\theta - \sin^2\theta \equiv 1 - 2\sin^2\theta \equiv 2\cos^2\theta - 1$$
$$\tan 2\theta \equiv \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Formulæ

$$\sin\frac{\theta}{2} \equiv \pm\sqrt{\frac{1 - \cos\theta}{2}}$$
$$\cos\frac{\theta}{2} \equiv \pm\sqrt{\frac{1 + \cos\theta}{2}}$$
$$\tan\frac{\theta}{2} \equiv \frac{\sin\theta}{1 + \cos\theta} \equiv \frac{1 - \cos\theta}{\sin\theta}$$

Products of Sines and Cosines

$$\sin\alpha\ \cos\beta \equiv \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

$$\cos\alpha\ \sin\beta \equiv \frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$$

$$\cos\alpha\ \cos\beta \equiv \frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

$$\sin\alpha\ \sin\beta \equiv -\frac{1}{2}\left[\cos(\alpha+\beta)-\cos(\alpha-\beta)\right]$$

Sums and Differences of Sines and Cosines

$$\sin\alpha+\sin\beta \equiv 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta \equiv 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha+\cos\beta \equiv 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta \equiv -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

Other Formulæ

$$\sin^2\theta \equiv \frac{1}{2}\left(1-\cos 2\theta\right)$$

$$\cos^2\theta \equiv \frac{1}{2}\left(1+\cos 2\theta\right)$$

$$\tan\theta \equiv \cot\theta - 2\cot 2\theta$$

Appendix D

Useful Series

The first four series are valid if $|x| < 1$; the last three are valid for all real x.

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \frac{7}{256}x^5 - \frac{21}{1024}x^6 + \frac{33}{2048}x^7 - \frac{429}{32768}x^8 + \cdots \quad \text{(D.1)}$$

$$(1-x)^{1/2} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \frac{5}{128}x^4 - \frac{7}{256}x^5 - \frac{21}{1024}x^6 - \frac{33}{2048}x^7 - \frac{429}{32768}x^8 - \cdots \quad \text{(D.2)}$$

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \frac{35}{128}x^4 - \frac{63}{256}x^5 + \frac{231}{1024}x^6 - \frac{429}{2048}x^7 + \frac{6435}{32768}x^8 - \cdots \quad \text{(D.3)}$$

$$(1-x)^{-1/2} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4 + \frac{63}{256}x^5 + \frac{231}{1024}x^6 + \frac{429}{2048}x^7 + \frac{6435}{32768}x^8 + \cdots \quad \text{(D.4)}$$

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 + \frac{1}{40320}x^8 + \frac{1}{362880}x^9 + \cdots \quad \text{(D.5)}$$

$$\sin x = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362880}x^9 - \frac{1}{39916800}x^{11} + \frac{1}{6227020800}x^{13} - \cdots \quad \text{(D.6)}$$

$$\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{40320}x^8 - \frac{1}{3628800}x^{10} + \frac{1}{479001600}x^{12} - \cdots \quad \text{(D.7)}$$

Appendix E

SI Units

Table E-1. SI base units.

Name	Symbol	Quantity
meter	m	length
kilogram	kg	mass
second	s	time
ampere	A	electric current
kelvin	K	temperature
mole	mol	amount of substance
candela	cd	luminous intensity

Table E-2. Derived SI units.

Name	Symbol	Definition	Base Units	Quantity
radian	rad	m / m	—	plane angle
steradian	sr	m^2 / m^2	—	solid angle
newton	N	kg m s^{-2}	kg m s^{-2}	force
joule	J	N m	kg m^2 s^{-2}	energy
watt	W	J / s	kg m^2 s^{-3}	power
pascal	Pa	N / m^2	kg m^{-1} s^{-2}	pressure
hertz	Hz	s^{-1}	s^{-1}	frequency
coulomb	C	A s	A s	electric charge
volt	V	J / C	kg m^2 A^{-1} s^{-3}	electric potential
ohm	Ω	V / A	kg m^2 A^{-2} s^{-3}	electrical resistance
siemens	S	A / V	kg^{-1} m^{-2} A^2 s^3	electrical conductance
farad	F	C / V	kg^{-1} m^{-2} A^2 s^4	capacitance
weber	Wb	V s	kg m^2 A^{-1} s^{-2}	magnetic flux
tesla	T	Wb / m^2	kg A^{-1} s^{-2}	magnetic induction
henry	H	Wb / A	kg m^2 A^{-2} s^{-2}	induction
lumen	lm	cd sr	cd sr	luminous flux
lux	lx	lm / m^2	cd sr m^{-2}	illuminance
becquerel	Bq	s^{-1}	s^{-1}	radioactivity
gray	Gy	J / kg	m^2 s^{-2}	absorbed dose
sievert	Sv	J / kg	m^2 s^{-2}	dose equivalent
katal	kat	mol / s	mol s^{-1}	catalytic activity

Table E-3. SI prefixes.

Prefix	Symbol	Definition	English
yotta-	Y	10^{24}	septillion
zetta-	Z	10^{21}	sextillion
exa-	E	10^{18}	quintillion
peta-	P	10^{15}	quadrillion
tera-	T	10^{12}	trillion
giga-	G	10^{9}	billion
mega-	M	10^{6}	million
kilo-	k	10^{3}	thousand
hecto-	h	10^{2}	hundred
deka-	da	10^{1}	ten
deci-	d	10^{-1}	tenth
centi-	c	10^{-2}	hundredth
milli-	m	10^{-3}	thousandth
micro-	μ	10^{-6}	millionth
nano-	n	10^{-9}	billionth
pico-	p	10^{-12}	trillionth
femto-	f	10^{-15}	quadrillionth
atto-	a	10^{-18}	quintillionth
zepto-	z	10^{-21}	sextillionth
yocto-	y	10^{-24}	septillionth

Table E-4. Prefixes for *computer use only*.

Prefix	Symbol	Definition	
yobi-	Yi	2^{80}	= 1,208,925,819,614,629,174,706,176
zebi-	Zi	2^{70}	= 1,180,591,620,717,411,303,424
exbi-	Ei	2^{60}	= 1,152,921,504,606,846,976
pebi-	Pi	2^{50}	= 1,125,899,906,842,624
tebi-	Ti	2^{40}	= 1,099,511,627,776
gibi-	Gi	2^{30}	= 1,073,741,824
mebi-	Mi	2^{20}	= 1,048,576
kibi-	Ki	2^{10}	= 1,024

Appendix F

Gaussian Units

Table F-1. Gaussian base units.

Name	Symbol	Quantity
centimeter	cm	length
gram	g	mass
second	s	time
kelvin	K	temperature
mole	mol	amount of substance
candela	cd	luminous intensity

Table F-2. Derived Gaussian units.

Name	Symbol	Definition	Base Units	Quantity
radian	rad	m / m	—	plane angle
steradian	sr	m^2 / m^2	—	solid angle
dyne	dyn	g cm s^{-2}	g cm s^{-2}	force
erg	erg	dyn cm	g cm^2 s^{-2}	energy
statwatt	statW	erg / s	g cm^2 s^{-3}	power
barye	ba	dyn / cm^2	g cm^{-1} s^{-2}	pressure
galileo	Gal	cm / s^2	cm s^{-2}	acceleration
poise	P	g / (cm s)	g cm^{-1} s^{-1}	dynamic viscosity
stokes	St	cm^2 / s	cm^2 s^{-1}	kinematic viscosity
hertz	Hz	s^{-1}	s^{-1}	frequency
statcoulomb	statC		$g^{1/2}$ $cm^{3/2}$ s^{-1}	electric charge
franklin	Fr	statC	$g^{1/2}$ $cm^{3/2}$ s^{-1}	electric charge
statampere	statA	statC / s	$g^{1/2}$ $cm^{3/2}$ s^{-2}	electric current
statvolt	statV	erg / statC	$g^{1/2}$ $cm^{1/2}$ s^{-1}	electric potential
statohm	statΩ	statV / statA	s cm^{-1}	electrical resistance
statfarad	statF	statC / statV	cm	capacitance
maxwell	Mx	statV cm	$g^{1/2}$ $cm^{3/2}$ s^{-1}	magnetic flux
gauss	G	Mx / cm^2	$g^{1/2}$ $cm^{-1/2}$ s^{-1}	magnetic induction
oersted	Oe	statA s / cm^2	$g^{1/2}$ $cm^{-1/2}$ s^{-1}	magnetic intensity
gilbert	Gb	statA	$g^{1/2}$ $cm^{3/2}$ s^{-2}	magnetomotive force
unit pole	pole	dyn / Oe	$g^{1/2}$ $cm^{3/2}$ s^{-1}	magnetic pole strength
stathenry	statH	erg / $statA^2$	s^2 cm^{-1}	induction
lumen	lm	cd sr	cd sr	luminous flux
phot	ph	lm / cm^2	cd sr cm^{-2}	illuminance
stilb	sb	cd / cm^2	cd cm^{-2}	luminance
lambert	Lb	$1/\pi$ cd / cm^2	cd cm^{-2}	luminance
kayser	K	1 / cm	cm^{-1}	wave number
becquerel	Bq	s^{-1}	s^{-1}	radioactivity
katal	kat	mol / s	mol s^{-1}	catalytic activity

Appendix G

Units of Physical Quantities

Table G-1. Units of physical quantities.

Quantity	SI Units	Gaussian Units
Absorbed dose	Gy	erg g^{-1}
Acceleration	m s^{-2}	cm s^{-2}
Amount of substance	mol	mol
Angle (plane)	rad	rad
Angle (solid)	sr	sr
Angular acceleration	rad s^{-2}	rad s^{-2}
Angular momentum	N m s	dyn cm s
Angular velocity	rad s^{-1}	rad s^{-1}
Area	m^2	cm^2
Bulk modulus	Pa	ba
Catalytic activity	kat	kat
Coercivity	A m^{-1}	Oe
Crackle	m s^{-5}	cm s^{-5}
Density	kg m^{-3}	g cm^{-3}
Distance	m	cm
Dose equivalent	Sv	erg g^{-1}
Elastic modulus	N m^{-2}	dyn cm^{-2}
Electric capacitance	F	statF
Electric charge	C	statC
Electric conductance	S	statΩ^{-1}
Electric conductivity	S m^{-1}	statΩ^{-1} cm^{-1}
Electric current	A	statA
Electric dipole moment	C m	statC cm
Electric displacement (D)	C m^{-2}	statC cm^{-2}
Electric elastance	F^{-1}	statF^{-1}
Electric field (E)	V m^{-1}	statV cm^{-1}
Electric flux	V m	statV cm
Electric permittivity	F m^{-1}	—
Electric polarization (P)	C m^{-2}	statC cm^{-2}
Electric potential	V	statV
Electric resistance	Ω	statΩ
Electric resistivity	Ω m	statΩ cm

Table G-1 (cont'd). Units of physical quantities.

Quantity	SI Units	Gaussian Units
Energy	J	erg
Enthalpy	J	erg
Entropy	J K^{-1}	erg K^{-1}
Force	N	dyn
Frequency	Hz	Hz
Heat	J	erg
Heat capacity	J K^{-1}	erg K^{-1}
Illuminance	lx	ph
Impulse	N s	dyn s
Inductance	H	statH
Jerk	m s^{-3}	cm s^{-3}
Jounce	m s^{-4}	cm s^{-4}
Latent heat	J kg^{-1}	erg g^{-1}
Length	m	cm
Luminance	cd m^{-2}	sb
Luminous flux	lm	lm
Luminous intensity	cd	cd
Magnetic flux	Wb	Mx
Magnetic induction (B)	T	G
Magnetic intensity (H)	A m^{-1}	Oe
Magnetic dipole moment (B convention)	A m^2	pole cm
Magnetic dipole moment (H convention)	Wb m	pole cm
Magnetic permeability	H m^{-1}	—
Magnetic permeance	H	s
Magnetic pole strength (B convention)	A m	unit pole
Magnetic pole strength (H convention)	Wb	unit pole
Magnetic potential (scalar)	A	Oe cm
Magnetic potential (vector)	T m	G cm
Magnetic reluctance	H^{-1}	s^{-1}
Magnetization (M)	A m^{-1}	Mx cm^{-2}
Magnetomotive force	A	Gb
Mass	kg	g
Memristance	Ω	statΩ
Molality	mol kg^{-1}	mol g^{-1}
Molarity	mol m^{-3}	mol cm^{-3}
Moment of inertia	kg m^2	g cm^2
Momentum	N s	dyn s
Pop	m s^{-6}	cm s^{-6}
Power	W	statW
Pressure	Pa	ba
Radioactivity	Bq	Bq
Remanence	T	G
Retentivity	T	G

Table G-1 (cont'd). Units of physical quantities.

Quantity	SI Units	Gaussian Units
Shear modulus	$N\ m^{-2}$	$dyn\ cm^{-2}$
Snap	$m\ s^{-4}$	$cm\ s^{-4}$
Specific heat	$J\ K^{-1}\ kg^{-1}$	$erg\ K^{-1}\ g^{-1}$
Strain	—	—
Stress	$N\ m^{-2}$	$dyn\ cm^{-2}$
Temperature	K	K
Tension	N	dyn
Time	s	s
Torque	N m	dyn cm
Velocity	$m\ s^{-1}$	$cm\ s^{-1}$
Viscosity (dynamic)	Pa s	P
Viscosity (kinematic)	$m^2\ s^{-1}$	St
Volume	m^3	cm^3
Wave number	m^{-1}	kayser
Weight	N	dyn
Work	J	erg
Young's modulus	$N\ m^{-2}$	$dyn\ cm^{-2}$

Appendix H

Physical Constants

Table H-1. Fundamental physical constants (CODATA 2010).

Description	Symbol	Value
Speed of light (vacuum)	c	2.99792458×10^{8} m/s
Gravitational constant	G	6.67384×10^{-11} m^3 kg^{-1} s^{-2}
Elementary charge	e	$1.602176565 \times 10^{-19}$ C
Permittivity of free space	ε_0	$8.85418781762038985 \ldots \times 10^{-12}$ F/m
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2
Coulomb constant $(1/(4\pi\varepsilon_0))$	k_c	$8.9875517873681764 \times 10^{9}$ m/F
Electron mass	m_e	$9.10938291 \times 10^{-31}$ kg
Proton mass	m_p	$1.672621777 \times 10^{-27}$ kg
Neutron mass	m_n	$1.674927351 \times 10^{-27}$ kg
Atomic mass unit (amu)	u	$1.660538921 \times 10^{-27}$ kg
Planck constant	h	$6.62606957 \times 10^{-34}$ J s
Planck constant $\div 2\pi$	$\hbar$	$1.054571726 \times 10^{-34}$ J s
Boltzmann constant	k_B	$1.3806488 \times 10^{-23}$ J/K

Table H-2. Other physical constants.

Description	Symbol	Value
Acceleration due to gravity at Earth surface	g	9.80 m/s^2
Radius of the Earth (eq.)	$R_\oplus$	6378.140 km
Mass of the Earth	$M_\oplus$	5.97320×10^{24} kg
Earth gravity constant	$GM_\oplus$	3.986005×10^{14} m^3 s^{-2}
Speed of sound in air (20°C)	$v_{\rm snd}$	343 m/s
Density of air (sea level)	$\rho_{\rm air}$	1.29 kg/m^3
Density of water	$\rho_{\rm w}$	1 g/cm^3 $=$ 1000 kg/m^3
Index of refraction of water	$n_{\rm w}$	1.33
Resistivity of copper (20°C)	$\rho_{\rm Cu}$	1.68×10^{-8} Ω m

Appendix I

Astronomical Data

Table I-1. Astronomical constants.

Description	Symbol	Value
Astronomical unit	AU	$1.49597870 \times 10^{11}$ m
Obliquity of ecliptic (J2000)	ε	$23^\circ.4392911$
Solar mass	$M_\odot$	1.9891×10^{30} kg
Solar radius	$R_\odot$	696,000 km

Table I-2. Planetary Data.

Planet	Mass (Yg)	Eq. radius (km)	Orbit semi-major axis (Gm)
Mercury	330.2	2439.7	57.91
Venus	4868.5	6051.8	108.21
Earth	5973.6	6378.1	149.60
Mars	641.85	3396.2	227.92
Jupiter	1,898,600	71,492	778.57
Saturn	568,460	60,268	1433.53
Uranus	86,832	25,559	2872.46
Neptune	102,430	24,764	4495.06
Pluto	12.5	1195	5906.38

Appendix J

Unit Conversion Tables

Time

1 day = 24 hours = 1440 minutes = 86400 seconds
1 hour = 60 minutes = 3600 seconds
1 year = 31 557 600 seconds $\approx \pi \times 10^7$ seconds

Length

1 mile = 8 furlongs = 80 chains = 320 rods = 1760 yards = 5280 feet = 1.609344 km
1 yard = 3 feet = 36 inches = 0.9144 meter
1 foot = 12 inches = 0.3048 meter
1 inch = 2.54 cm
1 nautical mile = 1852 meters = 1.15077944802354 miles
1 fathom = 6 feet
1 parsec = 3.26156376188 light-years = 206264.806245 AU = $3.08567756703 \times 10^{16}$ meters
1 ångström = 0.1 nm = 10^5 fermi = 10^{-10} meter

Mass

1 kilogram = 2.20462262184878 lb
1 pound = 16 oz = 0.45359237 kg
1 slug = 32.1740485564304 lb = 14.5939029372064 kg
1 short ton = 2000 lb
1 long ton = 2240 lb
1 metric ton = 1000 kg

Velocity

15 mph = 22 fps
1 mph = 0.44704 m/s
1 knot = 1.15077944802354 mph = 0.514444444444444 m/s

Area

1 acre = 43560 ft^2 = 4840 yd^2 = 4046.8564224 m^2
1 mile^2 = 640 acres = 2.589988110336 km^2
1 are = 100 m^2
1 hectare = 10^4 m^2 = 2.47105381467165 acres

Volume

1 liter = 1 dm^3 = 10^{-3} m^3 ≈ 1 quart
1 m^3 = 1000 liters
1 cm^3 = 1 mL
1 ft^3 = 1728 in^3 = 7.48051948051948 gal = 28.316846592 liters
1 gallon = 231 in^3 = 4 quarts = 8 pints = 16 cups = 3.785411784 liters
1 cup = 8 floz = 16 tablespoons = 48 teaspoons
1 tablespoon = 3 teaspoons = 4 fluidrams
1 dry gallon = 268.8025 in^3 = 4.40488377086 liters
1 imperial gallon = 4.54609 liters
1 bushel = 4 pecks = 8 dry gallons

Density

1 g/cm^3 = 1000 kg/m^3 = 8.34540445201933 lb/gal = 1.043175556502416 lb/pint

Force

1 lbf = 4.44822161526050 newtons = 32.1740485564304 poundals
1 newton = 10^5 dynes

Energy

1 calorie = 4.1868 joules
1 BTU = 1055.05585262 joules
1 ft-lb = 1.35581794833140 joules
1 kW-hr = 3.6 MJ
1 eV = $1.602176565 \times 10^{-19}$ joules
1 joule = 10^7 ergs

Power

1 horsepower = 745.69987158227022 watts
1 statwatt = 1 abwatt = 1 erg/s = 10^{-7} watt

Angle

$\text{rad} = \text{deg} \times \frac{\pi}{180}$ $\text{deg} = \text{rad} \times \frac{180}{\pi}$
1 deg = 60 arcmin = 3600 arcsec

Temperature

$°C = (°F-32) \times \frac{5}{9}$ $°F = \left(°C\times\frac{9}{5}\right) + 32$
$K = °C + 273.15$
$°R = °F + 459.67$

Pressure

1 atm = 101325 Pa = 1.01325 bar = 1013.25 millibar = 760 torr
= 760 mmHg = 29.9212598425197 inHg = 14.6959487755134 psi
= 2116.21662367394 lb/ft^2 = 1.05810831183697 ton/ft^2
= 1013250 $dyne/cm^2$ = 1013250 barye

Electromagnetism

1 statcoulomb = $3.335640951981520 \times 10^{-10}$ coulomb
1 abcoulomb = 10 coulombs
1 statvolt = 299.792458 volts
1 abvolt = 10^{-8} volt
1 maxwell = 10^{-8} weber
1 gauss = 10^{-4} tesla
1 oersted = $250/\pi$ (= 79.5774715459477) A/m

Appendix K

Angular Measure

K.1 Plane Angle

The most common unit of measure for plane angle is the *degree* (°), which is 1/360 of a full circle. Therefore a circle is 360°, a semicircle is 180°, and a right angle is 90°.

A similar unit (seldom used nowadays) is a sort of "metric" angle called the *grad*, defined so that a right angle is 100 grads, and so a full circle is 400 grads.

The SI unit of plane angle is the *radian* (rad), which is defined to be the angle that subtends an arc length equal to the radius of the circle. By this definition, a full circle subtends an angle equal to the arc length of a full circle ($2\pi r$) divided by its radius r — and so a full circle is 2π radians.

Since a hemisphere is 180° or π radians, the conversion factors are:

$$\text{rad} = \frac{\pi}{180} \times \text{deg} \tag{K.1}$$

$$\text{deg} = \frac{180}{\pi} \times \text{rad} \tag{K.2}$$

Subunits of the Degree

For small angles, a degree may be subdivided into 60 *minutes* ($'$), and a minute into 60 *seconds* ($''$). Thus a minute is 1/60 degree, and a second is 1/3600 degree.[1] Angles smaller than 1 second are sometimes expressed as *milli-arcseconds* (1/1000 arcsecond).[2]

K.2 Solid Angle

A *solid angle* is the three-dimensional version of a plane angle, and is subtended by the vertex of a cone. The SI unit of solid angle is the *steradian* (sr), which is defined to be the solid angle that subtends an area equal to the square of the radius of a circle. By this definition, a full sphere subtends an area equal to the area of a sphere ($4\pi r^2$) divided by the square of its radius (r^2) — so a full sphere is 4π steradians, and a hemisphere is 2π steradians.

[1] Sometimes these units are called the *minute of arc* or *arcminute*, and the *second of arc* or *arcsecond* to distinguish them from the units of time that have the same name.

[2] In an old system (Ref. [?]), the second was further subdivided into 60 *thirds* ($'''$), the third into 60 *fourths* ($''''$), etc. Under this system, 1 milli-arcsecond is 3.6 fourths of arc. This system is no longer used, though; today the second of arc is simply subdivided into decimals (e.g. $32.86473''$).

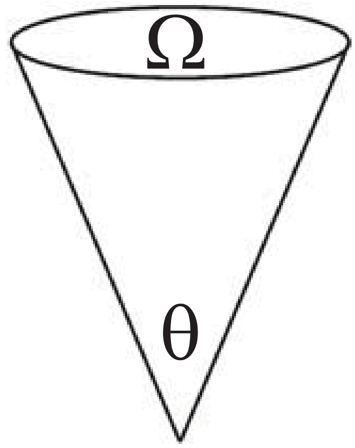

Figure K.1: Relation between plane angle θ and solid angle Ω for a right circular cone.

There is a simple relation between plane angle and solid angle for a right circular cone. If the vertex of the cone subtends an angle θ (the *aperture angle* of the cone), then the corresponding solid angle Ω is (Fig. K.1)

$$\Omega = 2\pi \left(1 - \cos\frac{\theta}{2}\right). \tag{K.3}$$

Another unit of solid angle is the *square degree* (deg^2):

$$\text{sq. deg.} = \text{sr} \times \left(\frac{180}{\pi}\right)^2. \tag{K.4}$$

In these units, a hemisphere is 20,626.48 deg^2, and a complete sphere is 41,252.96 deg^2.

Appendix L

The Gas Constant

Appendix M

Vector Arithmetic

A vector $\mathbf{A}$ may be written in cartesian (rectangular) form as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}, \tag{M.1}$$

where $\mathbf{i}$ is a *unit vector* (a vector of magnitude 1) in the x direction, $\mathbf{j}$ is a unit vector in the y direction, and $\mathbf{k}$ is a unit vector in the z direction. A_x, A_y, and A_z are called the x, y, and z *components* (respectively) of vector $\mathbf{A}$, and are the projections of the vector onto those axes.

The *magnitude* ("length") of vector $\mathbf{A}$ is

$$|\mathbf{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}. \tag{M.2}$$

For example, if $\mathbf{A} = 3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$, then $|\mathbf{A}| = A = \sqrt{3^2 + 5^2 + 2^2} = \sqrt{38}$.

In two dimensions, a vector has no $\mathbf{k}$ component: $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$.

Addition and Subtraction

To add two vectors, you add their components. Writing a second vector as $\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$, we have

$$\mathbf{A} + \mathbf{B} = (A_x + B_x)\,\mathbf{i} + (A_y + B_y)\,\mathbf{j} + (A_z + B_z)\,\mathbf{k}. \tag{M.3}$$

For example, if $\mathbf{A} = 3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$, then $\mathbf{A} + \mathbf{B} = 5\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$.

Subtraction of vectors is defined similarly:

$$\mathbf{A} - \mathbf{B} = (A_x - B_x)\,\mathbf{i} + (A_y - B_y)\,\mathbf{j} + (A_z - B_z)\,\mathbf{k}. \tag{M.4}$$

For example, if $\mathbf{A} = 3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$, then $\mathbf{A} - \mathbf{B} = \mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$.

Scalar Multiplication

To multiply a vector by a scalar, just multiply each component by the scalar. Thus if c is a scalar, then

$$c\mathbf{A} = cA_x\mathbf{i} + cA_y\mathbf{j} + cA_z\mathbf{k}. \tag{M.5}$$

For example, if $\mathbf{A} = 3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$, then $7\mathbf{A} = 21\mathbf{i} + 35\mathbf{j} + 14\mathbf{k}$.

Dot Product

It is possible to multiply a vector by another vector, but there is more than one kind of multiplication between vectors. One type of vector multiplication is called the *dot product*, in which a vector is multiplied by another vector to give a *scalar* result. The dot product (written with a dot operator, as in $\mathbf{A} \cdot \mathbf{B}$) is

$$\mathbf{A} \cdot \mathbf{B} = AB \cos\theta = A_x B_x + A_y B_y + A_z B_z, \tag{M.6}$$

where θ is the angle between vectors $\mathbf{A}$ and $\mathbf{B}$. For example, if $\mathbf{A} = 3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$, then $\mathbf{A} \cdot \mathbf{B} = 6 - 5 + 8 = 9$.

The dot product can be used to find the angle between two vectors. To do this, we solve Eq. (M.6) for θ and find $\cos\theta = \mathbf{A} \cdot \mathbf{B}/(AB)$. Applying this to the previous example, we get $A = \sqrt{38}$ and $B = \sqrt{21}$, so $\cos\theta = 9/(\sqrt{38}\sqrt{21})$, and thus $\theta = 71.4°$.

An immediate consequence of Eq. (M.6) is that two vectors are perpendicular if and only if their dot product is zero.

Cross Product

Another kind of multiplication between vectors, called the *cross product*, involves multiplying one vector by another and giving another *vector* as a result. The cross product is written with a cross operator, as in $\mathbf{A} \times \mathbf{B}$. It is defined by

$$\mathbf{A} \times \mathbf{B} = (AB \sin\theta)\, \mathbf{u} \tag{M.7}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \tag{M.8}$$

$$= (A_y B_z - A_z B_y)\, \mathbf{i} - (A_x B_z - A_z B_x)\, \mathbf{j} + (A_x B_y - A_y B_x)\, \mathbf{k}, \tag{M.9}$$

where again θ is the angle between the vectors, and $\mathbf{u}$ is a unit vector pointing in a direction perpendicular to the plane containing $\mathbf{A}$ and $\mathbf{B}$, in a right-hand sense: if you curl the fingers of your right hand from $\mathbf{A}$ into $\mathbf{B}$, then the thumb of your right hand points in the direction of $\mathbf{A} \times \mathbf{B}$ (Fig. M.1). As an example, if $\mathbf{A} = 3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$, then $\mathbf{A} \times \mathbf{B} = (20 - (-2))\mathbf{i} - (12 - 4)\mathbf{j} + (-3 - 10)\mathbf{k} = 22\mathbf{i} - 8\mathbf{j} - 13\mathbf{k}$.

Rectangular and Polar Forms

A two-dimensional vector may be written in either *rectangular form* $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$ described earlier, or in *polar form* $A\angle\theta$, where A is the vector magnitude, and θ is the direction measured counterclockwise from the $+x$ axis. To convert from polar form to rectangular form, one finds

$$A_x = A\cos\theta \tag{M.10}$$

$$A_y = A\sin\theta \tag{M.11}$$

Inverting these equations gives the expressions for converting from rectangular form to polar form:

$$A = \sqrt{A_x^2 + A_y^2} \tag{M.12}$$

$$\tan\theta = \frac{A_y}{A_x} \tag{M.13}$$

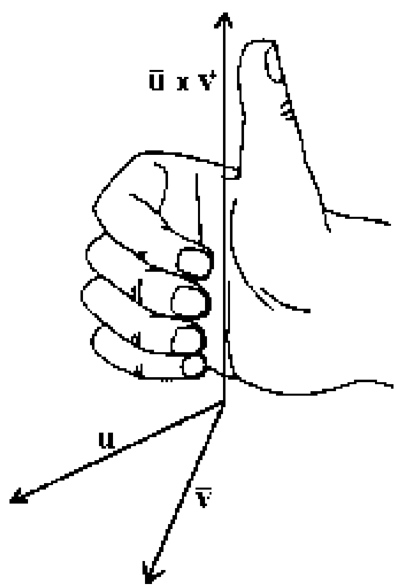

Figure M.1: The vector cross product $\mathbf{A} \times \mathbf{B}$ is perpendicular to the plane of $\mathbf{A}$ and $\mathbf{B}$, and in the right-hand sense. (*Credit: "Connected Curriculum Project", Duke University.*)

Appendix N

Matrix Properties

This appendix presents a brief summary of the properties of 2×2 and 3×3 matrices.

2×2 Matrices

Determinant

The determinant of a 2×2 matrix is given by the well-known formula:

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc. \tag{N.1}$$

Matrix of Cofactors

The matrix of cofactors is the matrix of signed minors; for a 2×2 matrix, this is

$$\text{cof} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} d & -c \\ -b & a \end{pmatrix} \tag{N.2}$$

Inverse

Finally, the inverse of a matrix is the transpose of the matrix of cofactors divided by the determinant. For a 2×2 matrix,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \tag{N.3}$$

3×3 Matrices

Determinant

The determinant of a 3×3 matrix is given by:

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg). \tag{N.4}$$

Matrix of Cofactors

The matrix of cofactors is the matrix of signed minors; for a 3×3 matrix, this is

$$\operatorname{cof} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = \begin{pmatrix} ei - fh & fg - di & dh - eg \\ ch - bi & ai - cg & bg - ah \\ bf - ce & cd - af & ae - bd \end{pmatrix} \tag{N.5}$$

Inverse

Finally, the inverse of a matrix is the transpose of the matrix of cofactors divided by the determinant. For a 3×3 matrix,

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^{-1} = \frac{1}{a(ei - fh) - b(di - fg) + c(dh - eg)} \begin{pmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{pmatrix} \tag{N.6}$$

Appendix O

TI-83+ Calculator Programs

Programs in this appendix are written for the Texas Instruments TI-83+ graphing calculator and similar models with the Z80 processor, using the built-in TI-BASIC language. Refer to Chapter 16 of the *TI-83+ Guidebook* for instructions on entering and running a program in the calculator.

O.1 Pendulum Period

Given the length L and amplitude θ of a simple plane pendulum, this program finds the period T, using the series expansion of Eq. (**??**) in Appendix **??**.

To run the program, execute program `PEND`. At the prompt `L=?` enter the pendulum length L in meters followed by `ENTER`. At the prompt `θ=?` enter the pendulum amplitude θ in degrees followed by `ENTER`. The program returns the period T in seconds.

After running the program, the calculator will be set to degrees mode.

Program Listing

```
PROGRAM:PEND
:Degree
:Prompt L,θ
:1→T
:For(N,1,34)
:T+(((2*N)!/(2^(2*N)*(N!)²))²)*(sin(0.5*θ))^(2*N)→T
:End
:2*π*√(L/9.8)*T→T
:Disp "T=",T
```

Example. Let $T = 1.2$ m and $\theta = 65°$. Enter the above program, press `PRGM` and execute program `PEND`. At the prompt `L=?` enter `1.2 ENTER`. At the prompt `θ=` enter `65 ENTER`. The program returns $T = 2.389769497$ sec.

Appendix P

TI Voyage 200 Calculator Programs

Programs in this appendix are written for the Texas Instruments Voyage 200 graphing calculator and similar models with the Motorola 68000 processor (TI-89 and TI-92), using the built-in TI-BASIC language. Refer to the "Programming" chapter of the *Voyage 200 Graphing Calculator* manual for instructions on entering and running a program in the calculator.

P.1 Pendulum Period

Given the length L and amplitude θ of a simple plane pendulum, this program finds the period T, using the series expansion of Eq. (**??**) in Appendix **??**.

To run the program, execute program `pend()`. At the prompt `l=?` enter the pendulum length L in meters followed by `ENTER`. At the prompt `θ=?` enter the pendulum amplitude θ in degrees followed by `ENTER`. The program returns the period T in seconds.

After running the program, the calculator will be set to degrees mode.

Program Listing

```
:pend()
:Prgm
:setMode("Angle","Degree")
:Prompt l,θ
:1→t
:For n,1,34
:  t+((2*n)!/(2^(2*n)*(n!)^2))^2*(sin(.5*θ))^(2*n)→t
:EndFor
:2*π*√(l/9.8)*t→t
:Disp "T=",t
:EndPrgm
```

Example. Let $T = 1.2$ m and $\theta = 65°$. Enter the above program, and execute program `pend()`. At the prompt `l=?` enter `1.2 ENTER`. At the prompt θ= enter `65 ENTER`. The program returns $T = 2.389769497$ sec.

Appendix Q

HP 35s / HP 15C Calculator Programs

Programs in this appendix are written for the Hewlett-Packard HP 35s and HP 15C scientific calculators, but can be easily modified to run on other HP calculators that use HP RPN.

Q.1 Pendulum Period

Given the length L and amplitude θ of a simple plane pendulum, this program finds the period T, using the series expansion of Eq. (**??**) in Appendix **??**.

To run the program, enter:

L	ENTER	θ	XEQ P	ENTER	(HP 35s)
L	ENTER	θ	f A		(HP 15C)

where L is in meters and θ is in degrees. The program returns the period T in seconds.

After running the program, the calculator will be set to degrees mode.

Program Listings

HP 35s		HP 15C	
P001	LBL P	001- 42,21,11	f LBL A
P002	DEG	002- 43 7	g DEG
P003	STO G	003- 44 .0	STO .0
P004	x<>y	004- 34	$x \gtrless y$
P005	STO L	005- 44 .1	STO .1
P006	1	006- 1	1
P007	STO T	007- 44 .2	STO .2
P008	1.034	008- 1	1
P009	STO K	009- 48	.
P010	RCL K	010- 0	0
P011	INTG	011- 3	3
P012	STO N	012- 4	4
P013	2	013- 44 .3	STO .3
P014	×	014- 42,21, 0	f LBL 0
P015	!	015- 45 .3	RCL .3
P016	2	016- 43 44	g INT

P017	RCL N	017-	44 .4	STO .4
P018	2	018-	2	2
P019	$\times$	019-	20	$\times$
P020	y^x	020-	42 0	f x!
P021	$\div$	021-	2	2
P022	RCL N	022-	45 .4	RCL .4
P023	!	023-	2	2
P024	x^2	024-	20	$\times$
P025	$\div$	025-	14	y^x
P026	x^2	026-	10	$\div$
P027	RCL G	027-	45 .4	RCL .4
P028	2	028-	42 0	f x!
P029	$\div$	029-	43 11	g x^2
P030	SIN	030-	10	$\div$
P031	RCL N	031-	43 11	g x^2
P032	2	032-	45 .0	RCL .0
P033	$\times$	033-	2	2
P034	y^x	034-	10	$\div$
P035	$\times$	035-	23	SIN
P036	STO+T	036-	45 .4	RCL .4
P037	ISG K	037-	2	2
P038	GTO P010	038-	20	$\times$
P039	RCL L	039-	14	y^x
P040	9.8	040-	20	$\times$
P041	$\div$	041-	44,40,.2	STO+.2
P042	$\sqrt{x}$	042-	42, 6,.3	f ISG .3
P043	2	043-	22 0	GTO 0
P044	$\times$	044-	45 .1	RCL .1
P045	π	045-	9	9
P046	$\times$	046-	48	.
P047	RCL T	047-	8	8
P048	$\times$	048-	10	$\div$
P049	RTN	049-	11	$\sqrt{x}$
		050-	2	2
		051-	20	$\times$
		052-	43 26	g π
		053-	20	$\times$
		054-	45 .2	RCL .2
		055-	20	$\times$
		056-	43 32	g RTN

Program: `LN=162 CK=B742`

Example. Let $L = 1.2$ m and $\theta = 65°$. Enter the above program, then type:

```
1.2   ENTER   65   XEQ P   ENTER      (HP 35S)
1.2   ENTER   65   f A                (HP 15C)
```

The program returns $T = 2.3898$ sec.

Appendix R

HP 50g Calculator Programs

Programs in this appendix are written for the Hewlett-Packard HP 50g scientific calculator and other HP calculators that use HP User RPL (e.g. the HP 48 series).

R.1 Pendulum Period

Given the length L and amplitude θ of a simple plane pendulum, this program finds the period T, using the series expansion of Eq. (**??**) in Appendix **??**.

After entering the program, store it into variable `PEND`. Then to run the program, enter: L `ENTER` θ `PEND`, where L is in meters and θ is in degrees. The program returns the period T in seconds.

After running the program, the calculator will be set to degrees mode.

Program Listing

```
≪ 1 → L,θ,T
   ≪ DEG
   1 34
   FOR N
      2 N * ! 2 2 N * ^ / N ! SQ / SQ θ 2 / SIN 2 N * ^ * T + 'T' STO
   NEXT
   L 9.8 / √ 2 * π * T * →NUM ≫
≫
```

Store the program into variable `PEND`.

Example. Let $L = 1.2$ m and $\theta = 65°$. Enter the above program, store into variable `PEND`, and type `1.2 ENTER 65 PEND`. The program returns $T = 2.3898$ sec.

Appendix S

Fundamental Physical Constants — Extensive Listing

The following tables, published by the National Institutes of Science and Technology (NIST), give the current best estimates of a large number of fundamental physical constants. These values were determined by the Committee on Data for Science and Technology (CODATA) for 2010, and are a best fit of the constants to the latest experimental results. (Source: `http://physics.nist.gov/constants`)

Fundamental Physical Constants — Extensive Listing

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
		UNIVERSAL		
speed of light in vacuum	c, c_0	299 792 458	m s^{-1}	exact
magnetic constant	μ_0	$4\pi \times 10^{-7}$	N A^{-2}	
		$= 12.566\,370\,614... \times 10^{-7}$	N A^{-2}	exact
electric constant $1/\mu_0 c^2$	ϵ_0	$8.854\,187\,817... \times 10^{-12}$	F m^{-1}	exact
characteristic impedance of vacuum $\mu_0 c$	Z_0	376.730 313 461...	Ω	exact
Newtonian constant of gravitation	G	$6.673\,84(80) \times 10^{-11}$	m^3 kg^{-1} s^{-2}	1.2×10^{-4}
	$G/\hbar c$	$6.708\,37(80) \times 10^{-39}$	$(\text{GeV}/c^2)^{-2}$	1.2×10^{-4}
Planck constant	h	$6.626\,069\,57(29) \times 10^{-34}$	J s	4.4×10^{-8}
		$4.135\,667\,516(91) \times 10^{-15}$	eV s	2.2×10^{-8}
$h/2\pi$	$\hbar$	$1.054\,571\,726(47) \times 10^{-34}$	J s	4.4×10^{-8}
		$6.582\,119\,28(15) \times 10^{-16}$	eV s	2.2×10^{-8}
	$\hbar c$	197.326 9718(44)	MeV fm	2.2×10^{-8}
Planck mass $(\hbar c/G)^{1/2}$	m_P	$2.176\,51(13) \times 10^{-8}$	kg	6.0×10^{-5}
energy equivalent	$m_\text{P} c^2$	$1.220\,932(73) \times 10^{19}$	GeV	6.0×10^{-5}
Planck temperature $(\hbar c^5/G)^{1/2}/k$	T_P	$1.416\,833(85) \times 10^{32}$	K	6.0×10^{-5}
Planck length $\hbar/m_\text{P} c = (\hbar G/c^3)^{1/2}$	l_P	$1.616\,199(97) \times 10^{-35}$	m	6.0×10^{-5}
Planck time $l_\text{P}/c = (\hbar G/c^5)^{1/2}$	t_P	$5.391\,06(32) \times 10^{-44}$	s	6.0×10^{-5}
		ELECTROMAGNETIC		
elementary charge	e	$1.602\,176\,565(35) \times 10^{-19}$	C	2.2×10^{-8}
	e/h	$2.417\,989\,348(53) \times 10^{14}$	A J^{-1}	2.2×10^{-8}
magnetic flux quantum $h/2e$	Φ_0	$2.067\,833\,758(46) \times 10^{-15}$	Wb	2.2×10^{-8}
conductance quantum $2e^2/h$	G_0	$7.748\,091\,7346(25) \times 10^{-5}$	S	3.2×10^{-10}
inverse of conductance quantum	G_0^{-1}	12 906.403 7217(42)	Ω	3.2×10^{-10}
Josephson constant[1] $2e/h$	K_J	$483\,597.870(11) \times 10^9$	Hz V^{-1}	2.2×10^{-8}
von Klitzing constant[2] $h/e^2 = \mu_0 c/2\alpha$	R_K	25 812.807 4434(84)	Ω	3.2×10^{-10}
Bohr magneton $e\hbar/2m_\text{e}$	μ_B	$927.400\,968(20) \times 10^{-26}$	J T^{-1}	2.2×10^{-8}
		$5.788\,381\,8066(38) \times 10^{-5}$	eV T^{-1}	6.5×10^{-10}
	μ_B/h	$13.996\,245\,55(31) \times 10^9$	Hz T^{-1}	2.2×10^{-8}
	μ_B/hc	46.686 4498(10)	m^{-1} T^{-1}	2.2×10^{-8}
	μ_B/k	0.671 713 88(61)	K T^{-1}	9.1×10^{-7}
nuclear magneton $e\hbar/2m_\text{p}$	μ_N	$5.050\,783\,53(11) \times 10^{-27}$	J T^{-1}	2.2×10^{-8}
		$3.152\,451\,2605(22) \times 10^{-8}$	eV T^{-1}	7.1×10^{-10}
	μ_N/h	7.622 593 57(17)	MHz T^{-1}	2.2×10^{-8}
	μ_N/hc	$2.542\,623\,527(56) \times 10^{-2}$	m^{-1} T^{-1}	2.2×10^{-8}
	μ_N/k	$3.658\,2682(33) \times 10^{-4}$	K T^{-1}	9.1×10^{-7}
		ATOMIC AND NUCLEAR		
		General		
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,5698(24) \times 10^{-3}$		3.2×10^{-10}
inverse fine-structure constant	α^{-1}	137.035 999 074(44)		3.2×10^{-10}
Rydberg constant $\alpha^2 m_\text{e} c/2h$	R_∞	10 973 731.568 539(55)	m^{-1}	5.0×10^{-12}
	$R_\infty c$	$3.289\,841\,960\,364(17) \times 10^{15}$	Hz	5.0×10^{-12}
	$R_\infty hc$	$2.179\,872\,171(96) \times 10^{-18}$	J	4.4×10^{-8}
		13.605 692 53(30)	eV	2.2×10^{-8}
Bohr radius $\alpha/4\pi R_\infty = 4\pi\epsilon_0\hbar^2/m_\text{e}e^2$	a_0	$0.529\,177\,210\,92(17) \times 10^{-10}$	m	3.2×10^{-10}
Hartree energy $e^2/4\pi\epsilon_0 a_0 = 2R_\infty hc = \alpha^2 m_\text{e} c^2$	E_h	$4.359\,744\,34(19) \times 10^{-18}$	J	4.4×10^{-8}
		27.211 385 05(60)	eV	2.2×10^{-8}
quantum of circulation	$h/2m_\text{e}$	$3.636\,947\,5520(24) \times 10^{-4}$	m^2 s^{-1}	6.5×10^{-10}

Fundamental Physical Constants — Extensive Listing

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
	h/m_e	$7.273\,895\,1040(47) \times 10^{-4}$	$m^2\ s^{-1}$	6.5×10^{-10}
		Electroweak		
Fermi coupling constant[3]	$G_F/(\hbar c)^3$	$1.166\,364(5) \times 10^{-5}$	GeV^{-2}	4.3×10^{-6}
weak mixing angle[4] θ_W (on-shell scheme)				
$\sin^2\theta_W = s^2_W \equiv 1-(m_W/m_Z)^2$	$\sin^2\theta_W$	$0.2223(21)$		9.5×10^{-3}
		Electron, e^-		
electron mass	m_e	$9.109\,382\,91(40) \times 10^{-31}$	kg	4.4×10^{-8}
		$5.485\,799\,0946(22) \times 10^{-4}$	u	4.0×10^{-10}
energy equivalent	$m_e c^2$	$8.187\,105\,06(36) \times 10^{-14}$	J	4.4×10^{-8}
		$0.510\,998\,928(11)$	MeV	2.2×10^{-8}
electron-muon mass ratio	m_e/m_μ	$4.836\,331\,66(12) \times 10^{-3}$		2.5×10^{-8}
electron-tau mass ratio	m_e/m_τ	$2.875\,92(26) \times 10^{-4}$		9.0×10^{-5}
electron-proton mass ratio	m_e/m_p	$5.446\,170\,2178(22) \times 10^{-4}$		4.1×10^{-10}
electron-neutron mass ratio	m_e/m_n	$5.438\,673\,4461(32) \times 10^{-4}$		5.8×10^{-10}
electron-deuteron mass ratio	m_e/m_d	$2.724\,437\,1095(11) \times 10^{-4}$		4.0×10^{-10}
electron-triton mass ratio	m_e/m_t	$1.819\,200\,0653(17) \times 10^{-4}$		9.1×10^{-10}
electron-helion mass ratio	m_e/m_h	$1.819\,543\,0761(17) \times 10^{-4}$		
electron to alpha particle mass ratio	m_e/m_α	$1.370\,933\,555\,78(55) \times 10^{-4}$		4.0×10^{-10}
electron charge to mass quotient	$-e/m_e$	$-1.758\,820\,088(39) \times 10^{11}$	$C\ kg^{-1}$	2.2×10^{-8}
electron molar mass $N_A m_e$	$M(e), M_e$	$5.485\,799\,0946(22) \times 10^{-7}$	$kg\ mol^{-1}$	4.0×10^{-10}
Compton wavelength $h/m_e c$	λ_C	$2.426\,310\,2389(16) \times 10^{-12}$	m	6.5×10^{-10}
$\lambda_C/2\pi = \alpha a_0 = \alpha^2/4\pi R_\infty$	$\lambdabar_C$	$386.159\,268\,00(25) \times 10^{-15}$	m	6.5×10^{-10}
classical electron radius $\alpha^2 a_0$	r_e	$2.817\,940\,3267(27) \times 10^{-15}$	m	9.7×10^{-10}
Thomson cross section $(8\pi/3)r_e^2$	σ_e	$0.665\,245\,8734(13) \times 10^{-28}$	m^2	1.9×10^{-9}
electron magnetic moment	μ_e	$-928.476\,430(21) \times 10^{-26}$	$J\ T^{-1}$	2.2×10^{-8}
to Bohr magneton ratio	μ_e/μ_B	$-1.001\,159\,652\,180\,76(27)$		2.6×10^{-13}
to nuclear magneton ratio	μ_e/μ_N	$-1838.281\,970\,90(75)$		4.1×10^{-10}
electron magnetic moment				
anomaly $\lvert\mu_e\rvert/\mu_B - 1$	a_e	$1.159\,652\,180\,76(27) \times 10^{-3}$		2.3×10^{-10}
electron g-factor $-2(1+a_e)$	g_e	$-2.002\,319\,304\,361\,53(53)$		2.6×10^{-13}
electron-muon magnetic moment ratio	μ_e/μ_μ	$206.766\,9896(52)$		2.5×10^{-8}
electron-proton magnetic moment ratio	μ_e/μ_p	$-658.210\,6848(54)$		8.1×10^{-9}
electron to shielded proton magnetic				
moment ratio (H_2O, sphere, 25 °C)	μ_e/μ'_p	$-658.227\,5971(72)$		1.1×10^{-8}
electron-neutron magnetic moment ratio	μ_e/μ_n	$960.920\,50(23)$		2.4×10^{-7}
electron-deuteron magnetic moment ratio	μ_e/μ_d	$-2143.923\,498(18)$		8.4×10^{-9}
electron to shielded helion magnetic				
moment ratio (gas, sphere, 25 °C)	μ_e/μ'_h	$864.058\,257(10)$		1.2×10^{-8}
electron gyromagnetic ratio $2\lvert\mu_e\rvert/\hbar$	γ_e	$1.760\,859\,708(39) \times 10^{11}$	$s^{-1}\ T^{-1}$	2.2×10^{-8}
	$\gamma_e/2\pi$	$28\,024.952\,66(62)$	$MHz\ T^{-1}$	2.2×10^{-8}
		Muon, μ^-		
muon mass	m_μ	$1.883\,531\,475(96) \times 10^{-28}$	kg	5.1×10^{-8}
		$0.113\,428\,9267(29)$	u	2.5×10^{-8}
energy equivalent	$m_\mu c^2$	$1.692\,833\,667(86) \times 10^{-11}$	J	5.1×10^{-8}
		$105.658\,3715(35)$	MeV	3.4×10^{-8}
muon-electron mass ratio	m_μ/m_e	$206.768\,2843(52)$		2.5×10^{-8}
muon-tau mass ratio	m_μ/m_τ	$5.946\,49(54) \times 10^{-2}$		9.0×10^{-5}
muon-proton mass ratio	m_μ/m_p	$0.112\,609\,5272(28)$		2.5×10^{-8}

Fundamental Physical Constants — Extensive Listing

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
muon-neutron mass ratio	m_μ/m_n	$0.112\,454\,5177(28)$		2.5×10^{-8}
muon molar mass $N_A m_\mu$	$M(\mu), M_\mu$	$0.113\,428\,9267(29)\times10^{-3}$	kg mol^{-1}	2.5×10^{-8}
muon Compton wavelength $h/m_\mu c$	$\lambda_{C,\mu}$	$11.734\,441\,03(30)\times10^{-15}$	m	2.5×10^{-8}
$\lambda_{C,\mu}/2\pi$	$\bar{\lambda}_{C,\mu}$	$1.867\,594\,294(47)\times10^{-15}$	m	2.5×10^{-8}
muon magnetic moment	μ_μ	$-4.490\,448\,07(15)\times10^{-26}$	J T^{-1}	3.4×10^{-8}
to Bohr magneton ratio	μ_μ/μ_B	$-4.841\,970\,44(12)\times10^{-3}$		2.5×10^{-8}
to nuclear magneton ratio	μ_μ/μ_N	$-8.890\,596\,97(22)$		2.5×10^{-8}
muon magnetic moment anomaly				
$\lvert\mu_\mu\rvert/(e\hbar/2m_\mu)-1$	a_μ	$1.165\,920\,91(63)\times10^{-3}$		5.4×10^{-7}
muon g-factor $-2(1+a_\mu)$	g_μ	$-2.002\,331\,8418(13)$		6.3×10^{-10}
muon-proton magnetic moment ratio	μ_μ/μ_p	$-3.183\,345\,107(84)$		2.6×10^{-8}
		Tau, τ^-		
tau mass[5]	m_τ	$3.167\,47(29)\times10^{-27}$	kg	9.0×10^{-5}
		$1.907\,49(17)$	u	9.0×10^{-5}
energy equivalent	$m_\tau c^2$	$2.846\,78(26)\times10^{-10}$	J	9.0×10^{-5}
		$1776.82(16)$	MeV	9.0×10^{-5}
tau-electron mass ratio	m_τ/m_e	$3477.15(31)$		9.0×10^{-5}
tau-muon mass ratio	m_τ/m_μ	$16.8167(15)$		9.0×10^{-5}
tau-proton mass ratio	m_τ/m_p	$1.893\,72(17)$		9.0×10^{-5}
tau-neutron mass ratio	m_τ/m_n	$1.891\,11(17)$		9.0×10^{-5}
tau molar mass $N_A m_\tau$	$M(\tau), M_\tau$	$1.907\,49(17)\times10^{-3}$	kg mol^{-1}	9.0×10^{-5}
tau Compton wavelength $h/m_\tau c$	$\lambda_{C,\tau}$	$0.697\,787(63)\times10^{-15}$	m	9.0×10^{-5}
$\lambda_{C,\tau}/2\pi$	$\bar{\lambda}_{C,\tau}$	$0.111\,056(10)\times10^{-15}$	m	9.0×10^{-5}
		Proton, p		
proton mass	m_p	$1.672\,621\,777(74)\times10^{-27}$	kg	4.4×10^{-8}
		$1.007\,276\,466\,812(90)$	u	8.9×10^{-11}
energy equivalent	$m_p c^2$	$1.503\,277\,484(66)\times10^{-10}$	J	4.4×10^{-8}
		$938.272\,046(21)$	MeV	2.2×10^{-8}
proton-electron mass ratio	m_p/m_e	$1836.152\,672\,45(75)$		4.1×10^{-10}
proton-muon mass ratio	m_p/m_μ	$8.880\,243\,31(22)$		2.5×10^{-8}
proton-tau mass ratio	m_p/m_τ	$0.528\,063(48)$		9.0×10^{-5}
proton-neutron mass ratio	m_p/m_n	$0.998\,623\,478\,26(45)$		4.5×10^{-10}
proton charge to mass quotient	e/m_p	$9.578\,833\,58(21)\times10^{7}$	C kg^{-1}	2.2×10^{-8}
proton molar mass $N_A m_p$	$M(\mathrm{p}), M_p$	$1.007\,276\,466\,812(90)\times10^{-3}$	kg mol^{-1}	8.9×10^{-11}
proton Compton wavelength $h/m_p c$	$\lambda_{C,p}$	$1.321\,409\,856\,23(94)\times10^{-15}$	m	7.1×10^{-10}
$\lambda_{C,p}/2\pi$	$\bar{\lambda}_{C,p}$	$0.210\,308\,910\,47(15)\times10^{-15}$	m	7.1×10^{-10}
proton rms charge radius	r_p	$0.8775(51)\times10^{-15}$	m	5.9×10^{-3}
proton magnetic moment	μ_p	$1.410\,606\,743(33)\times10^{-26}$	J T^{-1}	2.4×10^{-8}
to Bohr magneton ratio	μ_p/μ_B	$1.521\,032\,210(12)\times10^{-3}$		8.1×10^{-9}
to nuclear magneton ratio	μ_p/μ_N	$2.792\,847\,356(23)$		8.2×10^{-9}
proton g-factor $2\mu_p/\mu_N$	g_p	$5.585\,694\,713(46)$		8.2×10^{-9}
proton-neutron magnetic moment ratio	μ_p/μ_n	$-1.459\,898\,06(34)$		2.4×10^{-7}
shielded proton magnetic moment (H_2O, sphere, 25 °C)	μ'_p	$1.410\,570\,499(35)\times10^{-26}$	J T^{-1}	2.5×10^{-8}
to Bohr magneton ratio	μ'_p/μ_B	$1.520\,993\,128(17)\times10^{-3}$		1.1×10^{-8}
to nuclear magneton ratio	μ'_p/μ_N	$2.792\,775\,598(30)$		1.1×10^{-8}
proton magnetic shielding correction				
$1-\mu'_p/\mu_p$ (H_2O, sphere, 25 °C)	σ'_p	$25.694(14)\times10^{-6}$		5.3×10^{-4}

Fundamental Physical Constants — Extensive Listing

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
proton gyromagnetic ratio $2\mu_p/\hbar$	γ_p	$2.675\,222\,005(63) \times 10^8$	$s^{-1}\ T^{-1}$	2.4×10^{-8}
	$\gamma_p/2\pi$	$42.577\,4806(10)$	MHz T^{-1}	2.4×10^{-8}
shielded proton gyromagnetic ratio $2\mu'_p/\hbar$ (H_2O, sphere, 25 °C)	γ'_p	$2.675\,153\,268(66) \times 10^8$	$s^{-1}\ T^{-1}$	2.5×10^{-8}
	$\gamma'_p/2\pi$	$42.576\,3866(10)$	MHz T^{-1}	2.5×10^{-8}
Neutron, n				
neutron mass	m_n	$1.674\,927\,351(74) \times 10^{-27}$	kg	4.4×10^{-8}
		$1.008\,664\,916\,00(43)$	u	4.2×10^{-10}
energy equivalent	$m_n c^2$	$1.505\,349\,631(66) \times 10^{-10}$	J	4.4×10^{-8}
		$939.565\,379(21)$	MeV	2.2×10^{-8}
neutron-electron mass ratio	m_n/m_e	$1838.683\,6605(11)$		5.8×10^{-10}
neutron-muon mass ratio	m_n/m_μ	$8.892\,484\,00(22)$		2.5×10^{-8}
neutron-tau mass ratio	m_n/m_τ	$0.528\,790(48)$		9.0×10^{-5}
neutron-proton mass ratio	m_n/m_p	$1.001\,378\,419\,17(45)$		4.5×10^{-10}
neutron-proton mass difference	$m_n - m_p$	$2.305\,573\,92(76) \times 10^{-30}$	kg	3.3×10^{-7}
		$0.001\,388\,449\,19(45)$	u	3.3×10^{-7}
energy equivalent	$(m_n - m_p)c^2$	$2.072\,146\,50(68) \times 10^{-13}$	J	3.3×10^{-7}
		$1.293\,332\,17(42)$	MeV	3.3×10^{-7}
neutron molar mass $N_A m_n$	$M(n), M_n$	$1.008\,664\,916\,00(43) \times 10^{-3}$	kg mol^{-1}	4.2×10^{-10}
neutron Compton wavelength $h/m_n c$	$\lambda_{C,n}$	$1.319\,590\,9068(11) \times 10^{-15}$	m	8.2×10^{-10}
$\lambda_{C,n}/2\pi$	$\bar{\lambda}_{C,n}$	$0.210\,019\,415\,68(17) \times 10^{-15}$	m	8.2×10^{-10}
neutron magnetic moment	μ_n	$-0.966\,236\,47(23) \times 10^{-26}$	J T^{-1}	2.4×10^{-7}
to Bohr magneton ratio	μ_n/μ_B	$-1.041\,875\,63(25) \times 10^{-3}$		2.4×10^{-7}
to nuclear magneton ratio	μ_n/μ_N	$-1.913\,042\,72(45)$		2.4×10^{-7}
neutron g-factor $2\mu_n/\mu_N$	g_n	$-3.826\,085\,45(90)$		2.4×10^{-7}
neutron-electron magnetic moment ratio	μ_n/μ_e	$1.040\,668\,82(25) \times 10^{-3}$		2.4×10^{-7}
neutron-proton magnetic moment ratio	μ_n/μ_p	$-0.684\,979\,34(16)$		2.4×10^{-7}
neutron to shielded proton magnetic moment ratio (H_2O, sphere, 25 °C)	μ_n/μ'_p	$-0.684\,996\,94(16)$		2.4×10^{-7}
neutron gyromagnetic ratio $2\lvert\mu_n\rvert/\hbar$	γ_n	$1.832\,471\,79(43) \times 10^8$	$s^{-1}\ T^{-1}$	2.4×10^{-7}
	$\gamma_n/2\pi$	$29.164\,6943(69)$	MHz T^{-1}	2.4×10^{-7}
Deuteron, d				
deuteron mass	m_d	$3.343\,583\,48(15) \times 10^{-27}$	kg	4.4×10^{-8}
		$2.013\,553\,212\,712(77)$	u	3.8×10^{-11}
energy equivalent	$m_d c^2$	$3.005\,062\,97(13) \times 10^{-10}$	J	4.4×10^{-8}
		$1875.612\,859(41)$	MeV	2.2×10^{-8}
deuteron-electron mass ratio	m_d/m_e	$3670.482\,9652(15)$		4.0×10^{-10}
deuteron-proton mass ratio	m_d/m_p	$1.999\,007\,500\,97(18)$		9.2×10^{-11}
deuteron molar mass $N_A m_d$	$M(d), M_d$	$2.013\,553\,212\,712(77) \times 10^{-3}$	kg mol^{-1}	3.8×10^{-11}
deuteron rms charge radius	r_d	$2.1424(21) \times 10^{-15}$	m	9.8×10^{-4}
deuteron magnetic moment	μ_d	$0.433\,073\,489(10) \times 10^{-26}$	J T^{-1}	2.4×10^{-8}
to Bohr magneton ratio	μ_d/μ_B	$0.466\,975\,4556(39) \times 10^{-3}$		8.4×10^{-9}
to nuclear magneton ratio	μ_d/μ_N	$0.857\,438\,2308(72)$		8.4×10^{-9}
deuteron g-factor μ_d/μ_N	g_d	$0.857\,438\,2308(72)$		8.4×10^{-9}
deuteron-electron magnetic moment ratio	μ_d/μ_e	$-4.664\,345\,537(39) \times 10^{-4}$		8.4×10^{-9}
deuteron-proton magnetic moment ratio	μ_d/μ_p	$0.307\,012\,2070(24)$		7.7×10^{-9}
deuteron-neutron magnetic moment ratio	μ_d/μ_n	$-0.448\,206\,52(11)$		2.4×10^{-7}
Triton, t				

Fundamental Physical Constants — Extensive Listing

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
triton mass	m_t	$5.007\,356\,30(22) \times 10^{-27}$	kg	4.4×10^{-8}
		$3.015\,500\,7134(25)$	u	8.2×10^{-10}
energy equivalent	$m_t c^2$	$4.500\,387\,41(20) \times 10^{-10}$	J	4.4×10^{-8}
		$2808.921\,005(62)$	MeV	2.2×10^{-8}
triton-electron mass ratio	m_t/m_e	$5496.921\,5267(50)$		9.1×10^{-10}
triton-proton mass ratio	m_t/m_p	$2.993\,717\,0308(25)$		8.2×10^{-10}
triton molar mass $N_A m_t$	$M(t), M_t$	$3.015\,500\,7134(25) \times 10^{-3}$	kg mol^{-1}	8.2×10^{-10}
triton magnetic moment	μ_t	$1.504\,609\,447(38) \times 10^{-26}$	J T^{-1}	2.6×10^{-8}
to Bohr magneton ratio	μ_t/μ_B	$1.622\,393\,657(21) \times 10^{-3}$		1.3×10^{-8}
to nuclear magneton ratio	μ_t/μ_N	$2.978\,962\,448(38)$		1.3×10^{-8}
triton g-factor $2\mu_t/\mu_N$	g_t	$5.957\,924\,896(76)$		1.3×10^{-8}
		Helion, h		
helion mass	m_h	$5.006\,412\,34(22) \times 10^{-27}$	kg	4.4×10^{-8}
		$3.014\,932\,2468(25)$	u	8.3×10^{-10}
energy equivalent	$m_h c^2$	$4.499\,539\,02(20) \times 10^{-10}$	J	4.4×10^{-8}
		$2808.391\,482(62)$	MeV	2.2×10^{-8}
helion-electron mass ratio	m_h/m_e	$5495.885\,2754(50)$		9.2×10^{-10}
helion-proton mass ratio	m_h/m_p	$2.993\,152\,6707(25)$		8.2×10^{-10}
helion molar mass $N_A m_h$	$M(h), M_h$	$3.014\,932\,2468(25) \times 10^{-3}$	kg mol^{-1}	8.3×10^{-10}
helion magnetic moment	μ_h	$-1.074\,617\,486(27) \times 10^{-26}$	J T^{-1}	2.5×10^{-8}
to Bohr magneton ratio	μ_h/μ_B	$-1.158\,740\,958(14) \times 10^{-3}$		1.2×10^{-8}
to nuclear magneton ratio	μ_h/μ_N	$-2.127\,625\,306(25)$		1.2×10^{-8}
helion g-factor $2\mu_h/\mu_N$	g_h	$-4.255\,250\,613(50)$		1.2×10^{-8}
shielded helion magnetic moment (gas, sphere, 25 °C)	μ'_h	$-1.074\,553\,044(27) \times 10^{-26}$	J T^{-1}	2.5×10^{-8}
to Bohr magneton ratio	μ'_h/μ_B	$-1.158\,671\,471(14) \times 10^{-3}$		1.2×10^{-8}
to nuclear magneton ratio	μ'_h/μ_N	$-2.127\,497\,718(25)$		1.2×10^{-8}
shielded helion to proton magnetic moment ratio (gas, sphere, 25 °C)	μ'_h/μ_p	$-0.761\,766\,558(11)$		1.4×10^{-8}
shielded helion to shielded proton magnetic moment ratio (gas/H_2O, spheres, 25 °C)	μ'_h/μ'_p	$-0.761\,786\,1313(33)$		4.3×10^{-9}
shielded helion gyromagnetic ratio $2\|\mu'_h\|/\hbar$ (gas, sphere, 25 °C)	γ'_h	$2.037\,894\,659(51) \times 10^{8}$	s^{-1} T^{-1}	2.5×10^{-8}
	$\gamma'_h/2\pi$	$32.434\,100\,84(81)$	MHz T^{-1}	2.5×10^{-8}
		Alpha particle, α		
alpha particle mass	m_α	$6.644\,656\,75(29) \times 10^{-27}$	kg	4.4×10^{-8}
		$4.001\,506\,179\,125(62)$	u	1.5×10^{-11}
energy equivalent	$m_\alpha c^2$	$5.971\,919\,67(26) \times 10^{-10}$	J	4.4×10^{-8}
		$3727.379\,240(82)$	MeV	2.2×10^{-8}
alpha particle to electron mass ratio	m_α/m_e	$7294.299\,5361(29)$		4.0×10^{-10}
alpha particle to proton mass ratio	m_α/m_p	$3.972\,599\,689\,33(36)$		9.0×10^{-11}
alpha particle molar mass $N_A m_\alpha$	$M(\alpha), M_\alpha$	$4.001\,506\,179\,125(62) \times 10^{-3}$	kg mol^{-1}	1.5×10^{-11}
		PHYSICOCHEMICAL		
Avogadro constant	N_A, L	$6.022\,141\,29(27) \times 10^{23}$	mol^{-1}	4.4×10^{-8}
atomic mass constant $m_u = \frac{1}{12}m(^{12}C) = 1$ u	m_u	$1.660\,538\,921(73) \times 10^{-27}$	kg	4.4×10^{-8}
energy equivalent	$m_u c^2$	$1.492\,417\,954(66) \times 10^{-10}$	J	4.4×10^{-8}
		$931.494\,061(21)$	MeV	2.2×10^{-8}

Fundamental Physical Constants — Extensive Listing

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
Faraday constant[6] $N_A e$	F	$96\,485.3365(21)$	C mol^{-1}	2.2×10^{-8}
molar Planck constant	$N_A h$	$3.990\,312\,7176(28) \times 10^{-10}$	J s mol^{-1}	7.0×10^{-10}
	$N_A hc$	$0.119\,626\,565\,779(84)$	J m mol^{-1}	7.0×10^{-10}
molar gas constant	R	$8.314\,4621(75)$	J mol^{-1} K^{-1}	9.1×10^{-7}
Boltzmann constant R/N_A	k	$1.380\,6488(13) \times 10^{-23}$	J K^{-1}	9.1×10^{-7}
		$8.617\,3324(78) \times 10^{-5}$	eV K^{-1}	9.1×10^{-7}
	k/h	$2.083\,6618(19) \times 10^{10}$	Hz K^{-1}	9.1×10^{-7}
	k/hc	$69.503\,476(63)$	m^{-1} K^{-1}	9.1×10^{-7}
molar volume of ideal gas RT/p				
$T = 273.15$ K, $p = 100$ kPa	V_m	$22.710\,953(21) \times 10^{-3}$	m^3 mol^{-1}	9.1×10^{-7}
Loschmidt constant N_A/V_m	n_0	$2.651\,6462(24) \times 10^{25}$	m^{-3}	9.1×10^{-7}
molar volume of ideal gas RT/p				
$T = 273.15$ K, $p = 101.325$ kPa	V_m	$22.413\,968(20) \times 10^{-3}$	m^3 mol^{-1}	9.1×10^{-7}
Loschmidt constant N_A/V_m	n_0	$2.686\,7805(24) \times 10^{25}$	m^{-3}	9.1×10^{-7}
Sackur-Tetrode (absolute entropy) constant[7]				
$\frac{5}{2} + \ln[(2\pi m_u k T_1/h^2)^{3/2} k T_1/p_0]$				
$T_1 = 1$ K, $p_0 = 100$ kPa	S_0/R	$-1.151\,7078(23)$		2.0×10^{-6}
$T_1 = 1$ K, $p_0 = 101.325$ kPa		$-1.164\,8708(23)$		1.9×10^{-6}
Stefan-Boltzmann constant				
$(\pi^2/60)k^4/\hbar^3 c^2$	σ	$5.670\,373(21) \times 10^{-8}$	W m^{-2} K^{-4}	3.6×10^{-6}
first radiation constant $2\pi hc^2$	c_1	$3.741\,771\,53(17) \times 10^{-16}$	W m^2	4.4×10^{-8}
first radiation constant for spectral radiance $2hc^2$	c_{1L}	$1.191\,042\,869(53) \times 10^{-16}$	W m^2 sr^{-1}	4.4×10^{-8}
second radiation constant hc/k	c_2	$1.438\,7770(13) \times 10^{-2}$	m K	9.1×10^{-7}
Wien displacement law constants				
$b = \lambda_{max} T = c_2/4.965\,114\,231...$	b	$2.897\,7721(26) \times 10^{-3}$	m K	9.1×10^{-7}
$b' = \nu_{max}/T = 2.821\,439\,372...\, c/c_2$	b'	$5.878\,9254(53) \times 10^{10}$	Hz K^{-1}	9.1×10^{-7}

[1] See the "Adopted values" table for the conventional value adopted internationally for realizing representations of the volt using the Josephson effect.

[2] See the "Adopted values" table for the conventional value adopted internationally for realizing representations of the ohm using the quantum Hall effect.

[3] Value recommended by the Particle Data Group (Nakamura, *et al.*, 2010).

[4] Based on the ratio of the masses of the W and Z bosons m_W/m_Z recommended by the Particle Data Group (Nakamura, *et al.*, 2010). The value for $\sin^2\theta_W$ they recommend, which is based on a particular variant of the modified minimal subtraction ($\overline{\text{MS}}$) scheme, is $\sin^2\hat{\theta}_W(M_Z) = 0.231\,22(15)$.

[5] This and all other values involving m_τ are based on the value of $m_\tau c^2$ in MeV recommended by the Particle Data Group (Nakamura, *et al.*, 2010), but with a standard uncertainty of 0.29 MeV rather than the quoted uncertainty of -0.26 MeV, $+0.29$ MeV.

[6] The helion, symbol h, is the nucleus of the ^{3}He atom.

[7] The numerical value of F to be used in coulometric chemical measurements is $96\,485.3401(48)$ [5.0×10^{-8}] when the relevant current is measured in terms of representations of the volt and ohm based on the Josephson and quantum Hall effects and the internationally adopted conventional values of the Josephson and von Klitzing constants K_{J-90} and R_{K-90} given in the "Adopted values" table.

[8] The entropy of an ideal monoatomic gas of relative atomic mass A_r is given by $S = S_0 + \frac{3}{2}R \ln A_r - R \ln(p/p_0) + \frac{5}{2}R \ln(T/\text{K})$.

Appendix T

Periodic Table of the Elements

IUPAC Periodic Table of the Elements

Key: atomic number / **Symbol** / name / standard atomic weight

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 **H** hydrogen [1.007; 1.009]																	2 **He** helium 4.003
3 **Li** lithium [6.938; 6.997]	4 **Be** beryllium 9.012											5 **B** boron [10.80; 10.83]	6 **C** carbon [12.00; 12.02]	7 **N** nitrogen [14.00; 14.01]	8 **O** oxygen [15.99; 16.00]	9 **F** fluorine 19.00	10 **Ne** neon 20.18
11 **Na** sodium 22.99	12 **Mg** magnesium 24.31											13 **Al** aluminium 26.98	14 **Si** silicon [28.08; 28.09]	15 **P** phosphorus 30.97	16 **S** sulfur [32.05; 32.08]	17 **Cl** chlorine [35.44; 35.46]	18 **Ar** argon 39.95
19 **K** potassium 39.10	20 **Ca** calcium 40.08	21 **Sc** scandium 44.96	22 **Ti** titanium 47.87	23 **V** vanadium 50.94	24 **Cr** chromium 52.00	25 **Mn** manganese 54.94	26 **Fe** iron 55.85	27 **Co** cobalt 58.93	28 **Ni** nickel 58.69	29 **Cu** copper 63.55	30 **Zn** zinc 65.38(2)	31 **Ga** gallium 69.72	32 **Ge** germanium 72.63	33 **As** arsenic 74.92	34 **Se** selenium 78.96(3)	35 **Br** bromine 79.90	36 **Kr** krypton 83.80
37 **Rb** rubidium 85.47	38 **Sr** strontium 87.62	39 **Y** yttrium 88.91	40 **Zr** zirconium 91.22	41 **Nb** niobium 92.91	42 **Mo** molybdenum 95.96(2)	43 **Tc** technetium	44 **Ru** ruthenium 101.1	45 **Rh** rhodium 102.9	46 **Pd** palladium 106.4	47 **Ag** silver 107.9	48 **Cd** cadmium 112.4	49 **In** indium 114.8	50 **Sn** tin 118.7	51 **Sb** antimony 121.8	52 **Te** tellurium 127.6	53 **I** iodine 126.9	54 **Xe** xenon 131.3
55 **Cs** caesium 132.9	56 **Ba** barium 137.3	57-71 lanthanoids	72 **Hf** hafnium 178.5	73 **Ta** tantalum 180.9	74 **W** tungsten 183.8	75 **Re** rhenium 186.2	76 **Os** osmium 190.2	77 **Ir** iridium 192.2	78 **Pt** platinum 195.1	79 **Au** gold 197.0	80 **Hg** mercury 200.6	81 **Tl** thallium [204.3; 204.4]	82 **Pb** lead 207.2	83 **Bi** bismuth 209.0	84 **Po** polonium	85 **At** astatine	86 **Rn** radon
87 **Fr** francium	88 **Ra** radium	89-103 actinoids	104 **Rf** rutherfordium	105 **Db** dubnium	106 **Sg** seaborgium	107 **Bh** bohrium	108 **Hs** hassium	109 **Mt** meitnerium	110 **Ds** darmstadtium	111 **Rg** roentgenium	112 **Cn** copernicium		114 **Fl** flerovium		116 **Lv** livermorium		

57 **La** lanthanum 138.9	58 **Ce** cerium 140.1	59 **Pr** praseodymium 140.9	60 **Nd** neodymium 144.2	61 **Pm** promethium	62 **Sm** samarium 150.4	63 **Eu** europium 152.0	64 **Gd** gadolinium 157.3	65 **Tb** terbium 158.9	66 **Dy** dysprosium 162.5	67 **Ho** holmium 164.9	68 **Er** erbium 167.3	69 **Tm** thulium 168.9	70 **Yb** ytterbium 173.1	71 **Lu** lutetium 175.0
89 **Ac** actinium	90 **Th** thorium 232.0	91 **Pa** protactinium 231.0	92 **U** uranium 238.0	93 **Np** neptunium	94 **Pu** plutonium	95 **Am** americium	96 **Cm** curium	97 **Bk** berkelium	98 **Cf** californium	99 **Es** einsteinium	100 **Fm** fermium	101 **Md** mendelevium	102 **No** nobelium	103 **Lr** lawrencium

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Notes

- IUPAC 2009 Standard atomic weights abridged to four significant digits (Table 4 published in *Pure Appl. Chem.* 83, 359-396 (2011); doi:10.1351/PAC-REP-10-09-14). The uncertainty in the last digit of the standard atomic weight value is listed in parentheses following the value. In the absence of parentheses, the uncertainty is one in that last digit. An interval in square brackets provides the lower and upper bounds of the standard atomic weight for that element. No values are listed for elements which lack isotopes with a characteristic isotopic abundance in natural terrestrial samples. See PAC for more details.

- "Aluminum" and "cesium" are commonly used alternative spellings for "aluminium" and "caesium."

- Claims for the discovery of all the remaining elements in the last row of the Table, namely elements with atomic numbers 113, 115, 117 and 118, and for which no assignments have yet been made, are being considered by a IUPAC and IUPAP Joint Working Party.

For updates to this table, see iupac.org/reports/periodic_table/. This version is dated 1 June 2012.

References

[1] Enrico Fermi, *Thermodynamics* (Dover, New York, 1956).

[2] H.C. van Ness, *Understanding Thermodynamics* (Dover, New York, 1983).

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